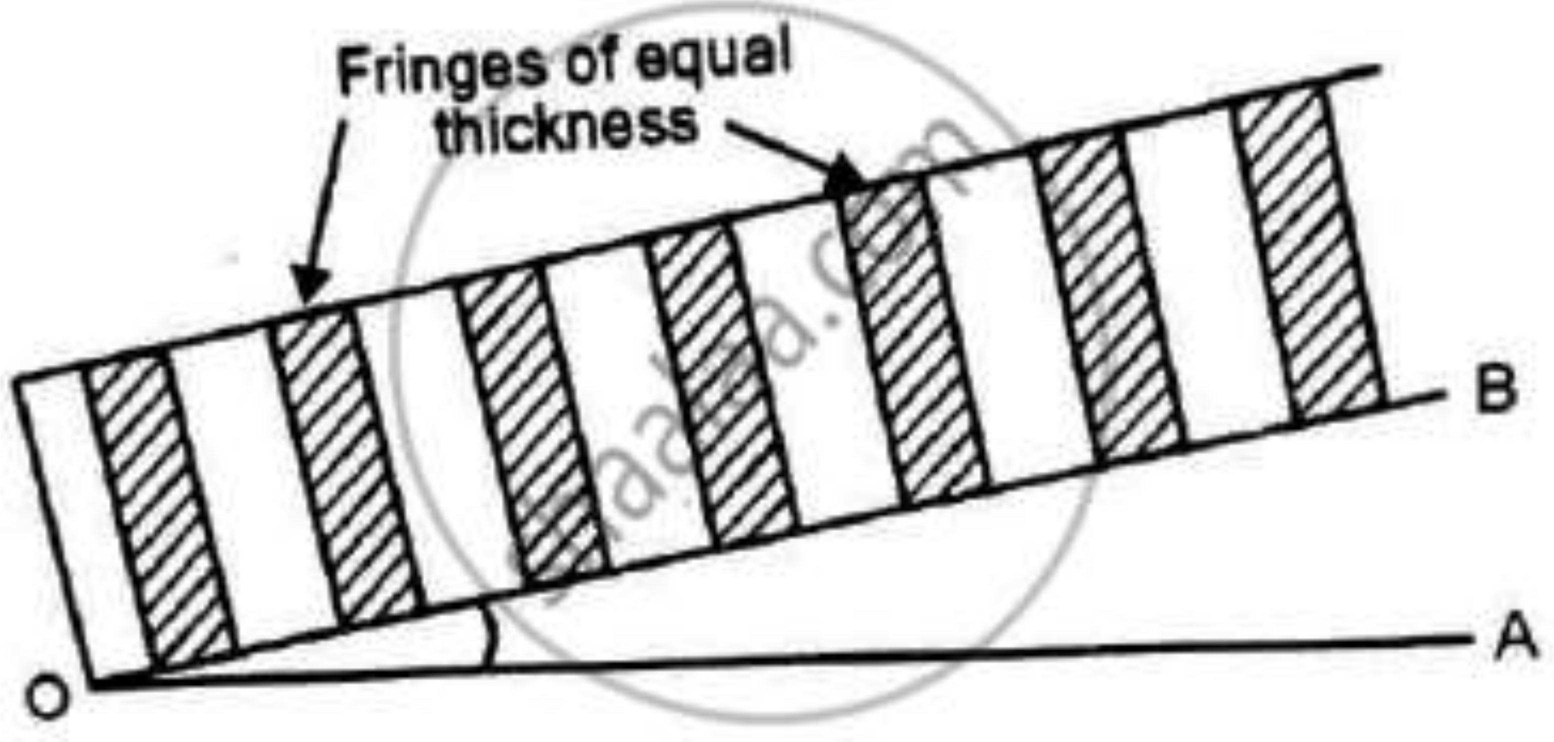
Interference

Air Wedge



8.22 TESTING THE PLANENESS OF SURFACES

If the two surfaces *OA* and *OB* are perfectly plane, the air-film gradually varies in thickness from *O* to *A*. The fringes are of equal thickness because each fringe is the locus of the points at which the thickness of the film has a constant value (Fig. 8.23).

This is an important application of the phenomenon of interference. If the fringes are not of equal thickness it means the surfaces are not plane. The standard method is to take an optically plane surface *OA* and the surface to be tested *OB*. The fringes are observed in the field of view and if they are of equal thickness the surface *OB* is plane. If not, the surface *OB* is not plane. The surface *OB* is polished and the process is repeated. When the fringes observed are of equal width, it means that the surface *OB* is plane.

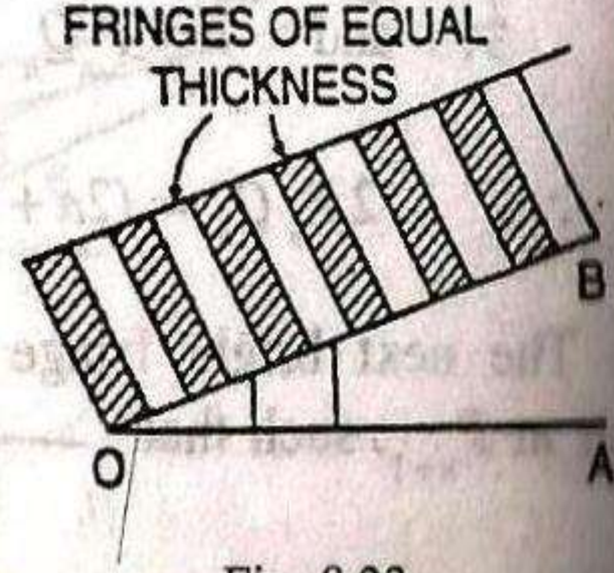


Fig. 8.23.

Newton’s Rings

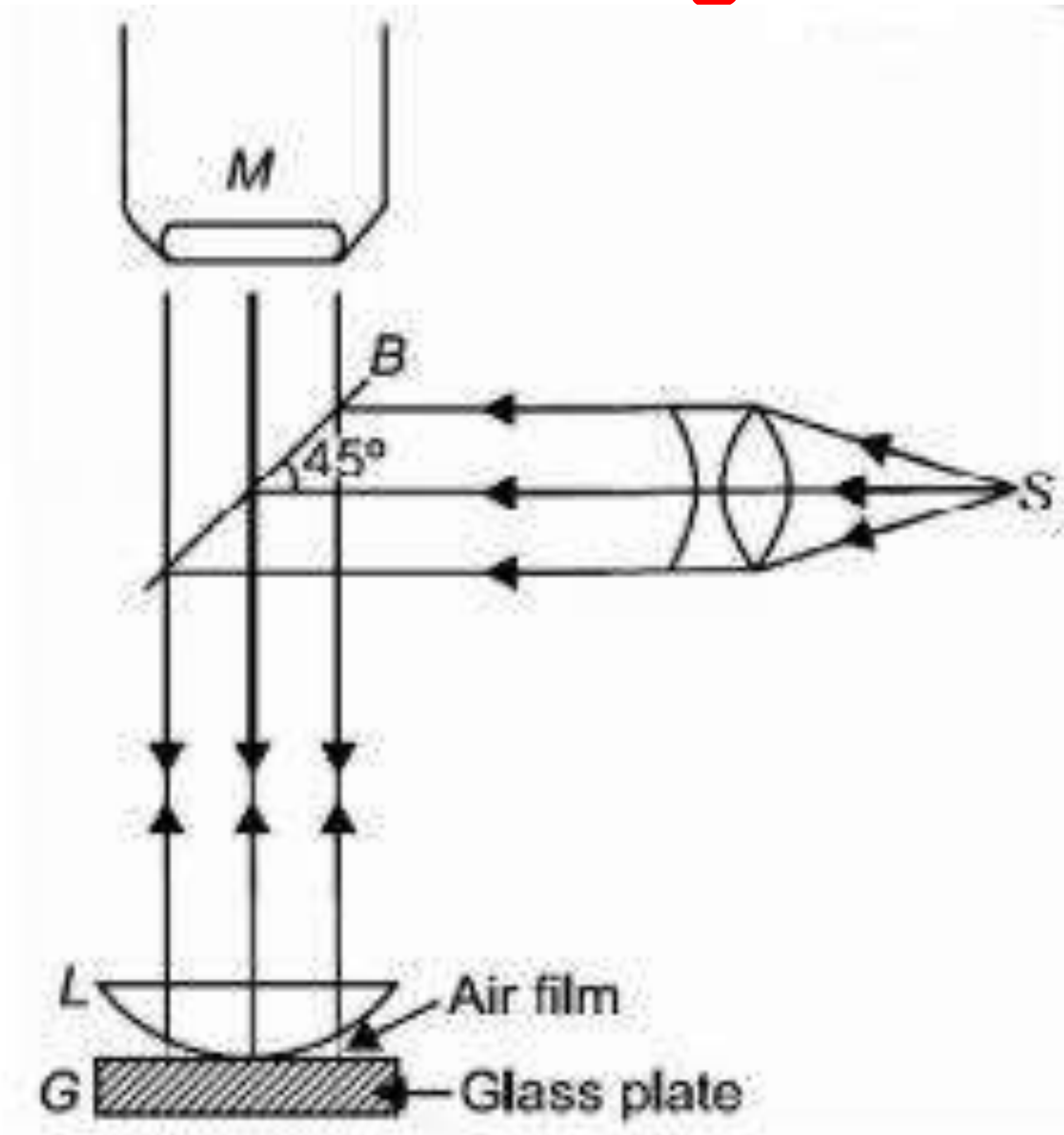


Fig. (i)

Circular interference fringes can be produced by enclosing a very thin film of air or any other transparent medium of varying thickness between a plane glass plate and a convex lens of a large radius of curvature. Such fringes were first obtained by Newton and are known as Newton’s rings.

When a plane-convex lens of long focal length is placed on a plane glass plate, a thin film of air is enclosed between the lower surface of the lens and the upper surface of the plate. The thickness of the air film is very small at the point of contact and gradually increases from the centre outwards. The fringes produced with monochromatic light are circular. The fringes are concentric circles, uniform in thickness and with the point of contact as the centre. When viewed with white light, the fringes are coloured. With monochromatic light, bright and dark circular fringes are produced in the air film.

S is a source of monochromatic light as shown in Fig.-(i). A horizontal beam of light falls on the glass plate B at 45°. The glass plate B reflects a part of the incident light towards the air film enclosed by the lens L and the plane glass plate G. The reflected beam from the air film is viewed with a microscope, Interference takes place and dark and bright circular fringes are produced. This is due to the interference between the light reflected from the lower surface of the lens and the upper surface of the glass plate G.

Theory:

(i) Newton's rings by reflected light:

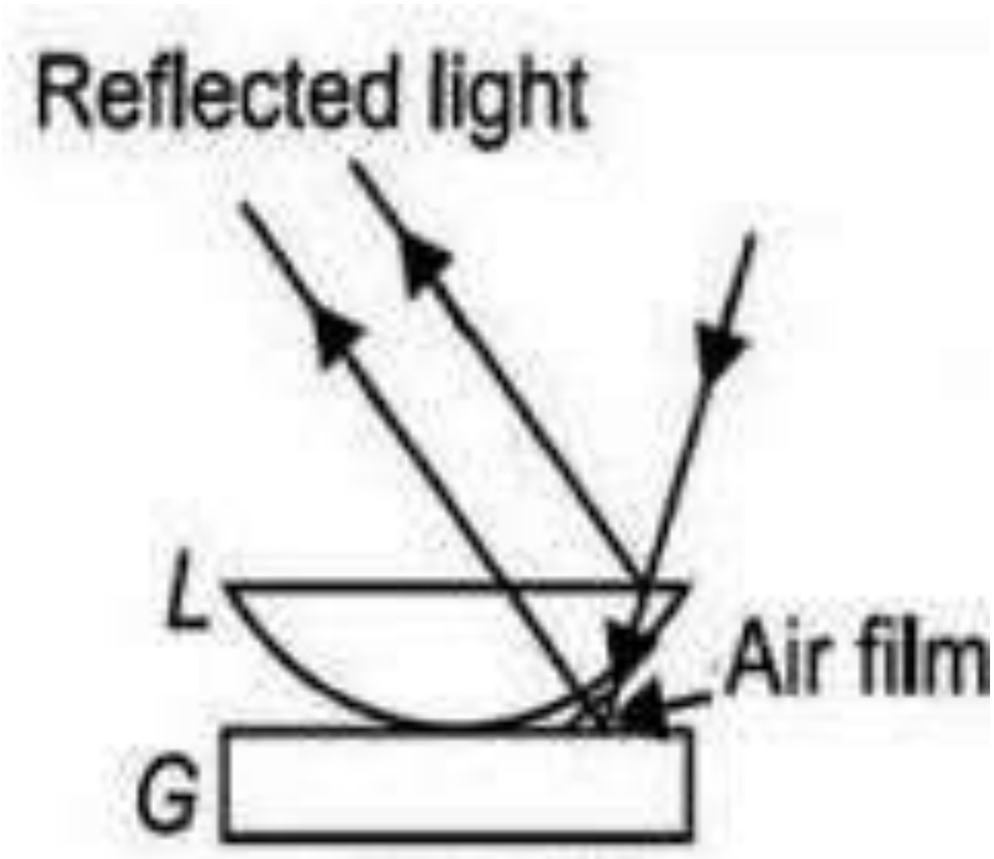


Fig. (ii)

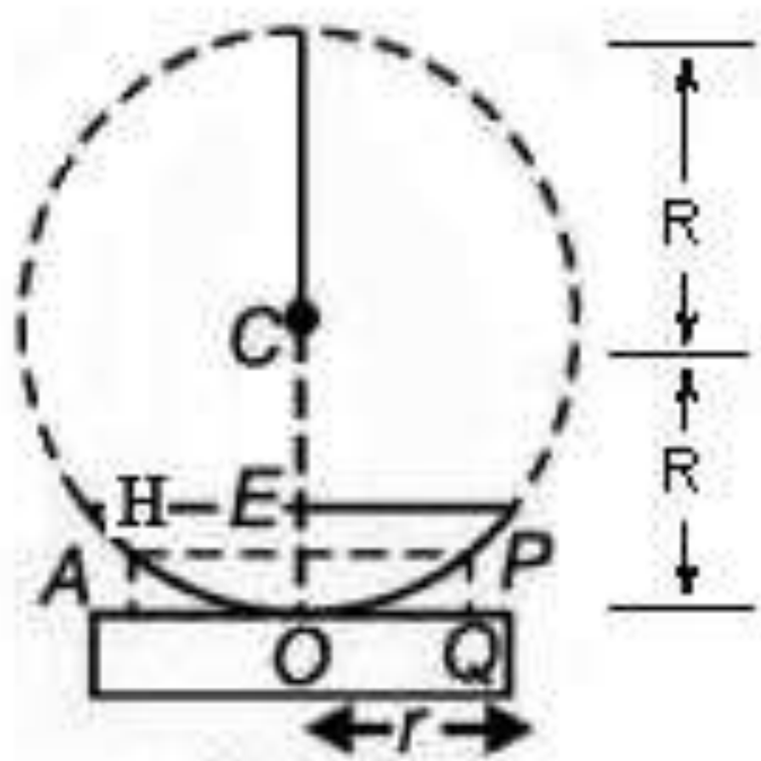


Fig. (iii)

Suppose the radius of curvature of the lens is R and the air film is of thickness t at a distance of OQ = r from the point of contact O. Here, interference is due to reflected light.

Therefore, for the bright rings

$$2\mu t \cos\theta = (2n-1)\frac{\lambda}{2} \quad \text{...(i)}$$

where $n = 1, 2, 3, \ldots$ etc.

Here, θ is small, therefore

$$\cos\theta = 1$$

For air, $\mu = 1$

$$2t = (2n-1)\frac{\lambda}{2} \quad \text{...(ii)}$$

For the dark rings

$$2\mu t \cos\theta = n\lambda$$

or

$$2t = n\lambda \quad \text{...(iii)}$$

where $n = 0, 1, 2, 3, \ldots\ldots$ etc.

In Fig **(i)**, $EP \times HE = OE \times (2R - OE)$

But $$EP = HE = r, \ OE = PQ = t$$

and $$2R - t = 2R \ \text{(Approximately)}$$

$$r^2 = 2R \cdot t$$

or $$t = \frac{r^2}{2R}$$

Substituting the value of t in equations (ii) and (iii).

For bright rings $$r^2 = \frac{(2n-1)\lambda R}{2}$$

$$\boxed{r = \sqrt{\frac{(2n-1)\lambda R}{2}}}$$

For dark rings

$$r^2 = n\lambda R$$

$$\boxed{r = \sqrt{n\lambda R}}$$

when $n = 0$, the radius of the dark ring is zero and the radius of the bright ring is $\sqrt{\frac{\lambda R}{2}}$. Therefore, the centre is dark. Alternately, dark and bright rings are produced.

Result: The radius of the dark ring is proportional to

(i) $\sqrt{n}$, (ii) $\sqrt{\lambda}$ and (iii) $\sqrt{R}$.

Similarly the radius of the bright ring is proportional to

(i) $\sqrt{\frac{(2n-1)}{2}}$, (ii) $\sqrt{\lambda}$ and (iii) $\sqrt{R}$.

If D is the diameter of the dark ring

$$D = 2r = 2\sqrt{n\lambda R}$$

For the central dark ring

$$n = 0$$

$$D = 2\sqrt{n\lambda R} = 0$$

This corresponds to the centre of the Newton's rings.

While counting the order of the dark rings 1, 2, 3 etc, the central ring is not counted.

Therefore for the first dark ring

$$n = 1$$

$$D_1 = 2\sqrt{\lambda R}$$

For the second dark ring

$$n = 2$$

$$D_2 = 2\sqrt{2\lambda R}$$

and for the n^{th} dark ring

$$D_n = 2\sqrt{n\lambda R}$$

Take the case of 16^{th} and 9^{th} rings

$$D_{16} = 2\sqrt{16\lambda R} = 8\sqrt{\lambda R}$$

$$D_9 = 2\sqrt{9\lambda R} = 6\sqrt{\lambda R}$$

The difference in diameters between the 16th and the 9th rings,

$$D_{16} - D_9 = 8\sqrt{\lambda R} - 6\sqrt{\lambda R} = 2\sqrt{\lambda R}$$

Similarly the difference in the diameters between the fourth and first rings,

$$D_4 - D_1 = 2\sqrt{4\lambda R} - 2\sqrt{\lambda R} = 2\sqrt{\lambda R}$$

Therefore, the fringe width decreases with the order of the fringe and the fringes get closer with increase in their order.

For bright rings,

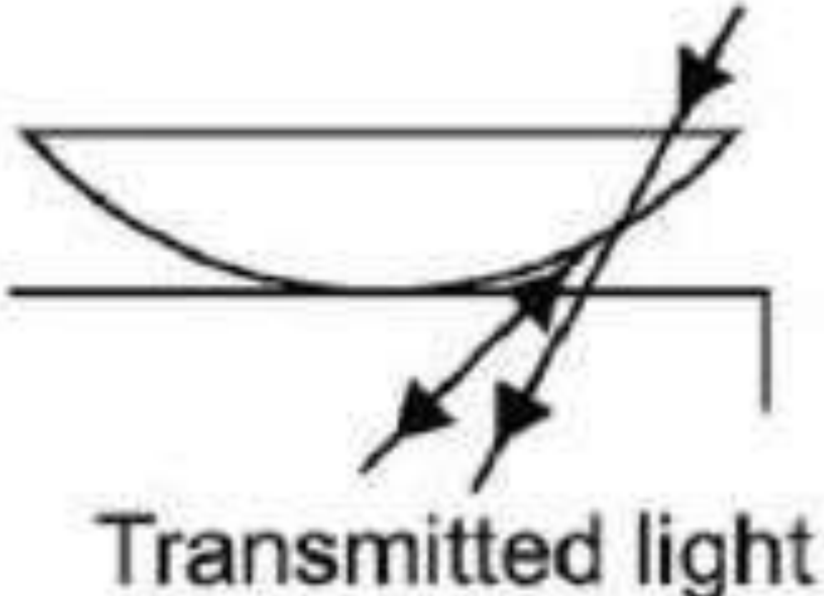
$$r_n^2 = \frac{(2n-1)\lambda R}{2}$$

$$D_n^2 = \frac{2(2n-1)\lambda R}{2}$$

$$r_n = \sqrt{\frac{(2n-1)\lambda R}{2}}$$

In above equation, substituting $n = 1, 2, 3$ (number of the ring) the radii of the first, second, third etc., bright rings can be obtained directly.

(ii) *Newton's rings by transmitted light:*



In the case of transmitted light, the interference fringes are produced such that for bright rings,

$$2\mu t \cos\theta = n\lambda$$

and for dark rings

$$2\mu t \cos\theta = (2n-1)\frac{\lambda}{2}$$

Here, for air $\mu = 1$, and $\cos\theta = 1$

For bright rings $\quad 2t = n\lambda$

and for dark rings $\quad 2t = (2n-1)\frac{\lambda}{2}$

Taking the value of $t = \frac{r^2}{2R}$, where r is the radius of the ring and R the radius of curvature of the lower surface of the lens, the radius for the bright and dark rings can be calculated.

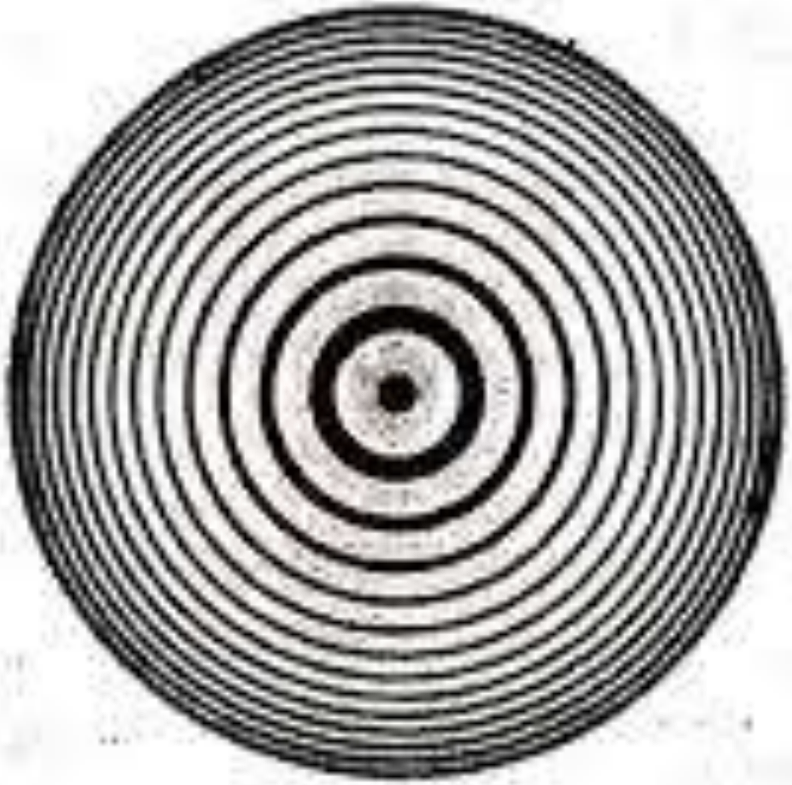
For bright rings, $r^2 = n\lambda R$

For dark rings, $r^2 = \frac{(2n-1)\lambda R}{2}$

where $n = 1, 2, 3, ...,$ etc.

When $n = 0$, for bright rings $r = 0$

Therefore, in the case of Newton's rings due to transmitted light, the central ring is bright i.e. just opposite to the ring pattern due to reflected light.



Newtons rings in reflected light

Newtons rings in transmitted light

8.29 HAIDINGER'S FRINGES

In the relation $2\,\mu t \cos r = n\lambda$, if t is large, a very small change in r will change the path difference by one wavelength. In this case the ray must pass through a plate as a parallel beam and must be received by the eye or the telescope focussed for infinity. The interference patterns are known as fringes of equal inclination. These are different from Newton's rings. These fringes of equal inclination were first observed by Haidinger and afterwards studied by Lummer and Mascart. From an extended source S, light rays fall on the plate. The rays striking at the same angle and refracted at the same angle form a parallel beam and are viewed through the telescope focussed for infinity (Fig. 8.35). The pattern is a series of concentric circles whose centre is the principal focus of the objective of the telescope.

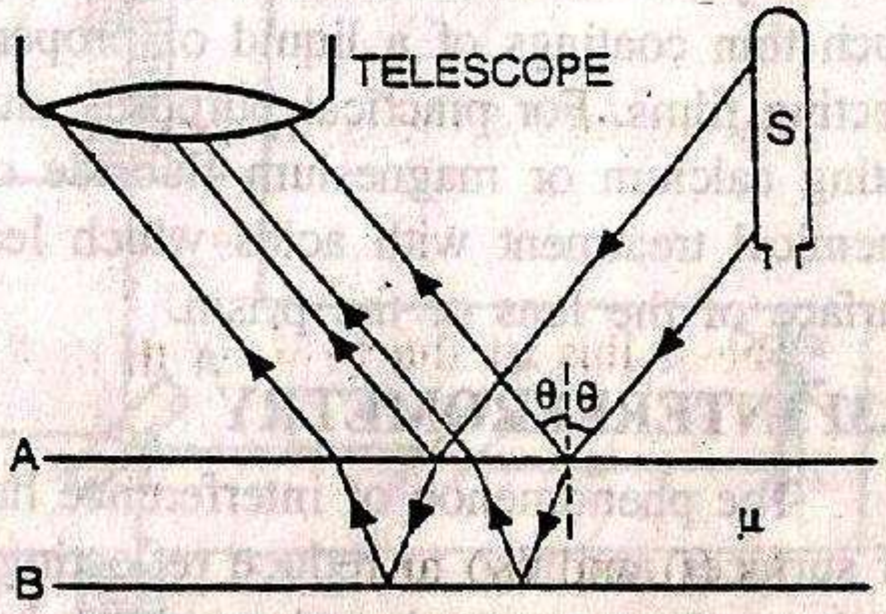


Fig. 8.35.

8.41 BREWSTER'S FRINGES

When a beam of monochromatic light falls in succession on two thick plates of transparent material, it is divided into several portions by reflection at various surfaces. Some of the reflected rays produce interference and interference fringes are observed. Such fringes were first observed by Brewster in 1815 and are known as **Brewster's fringes**.

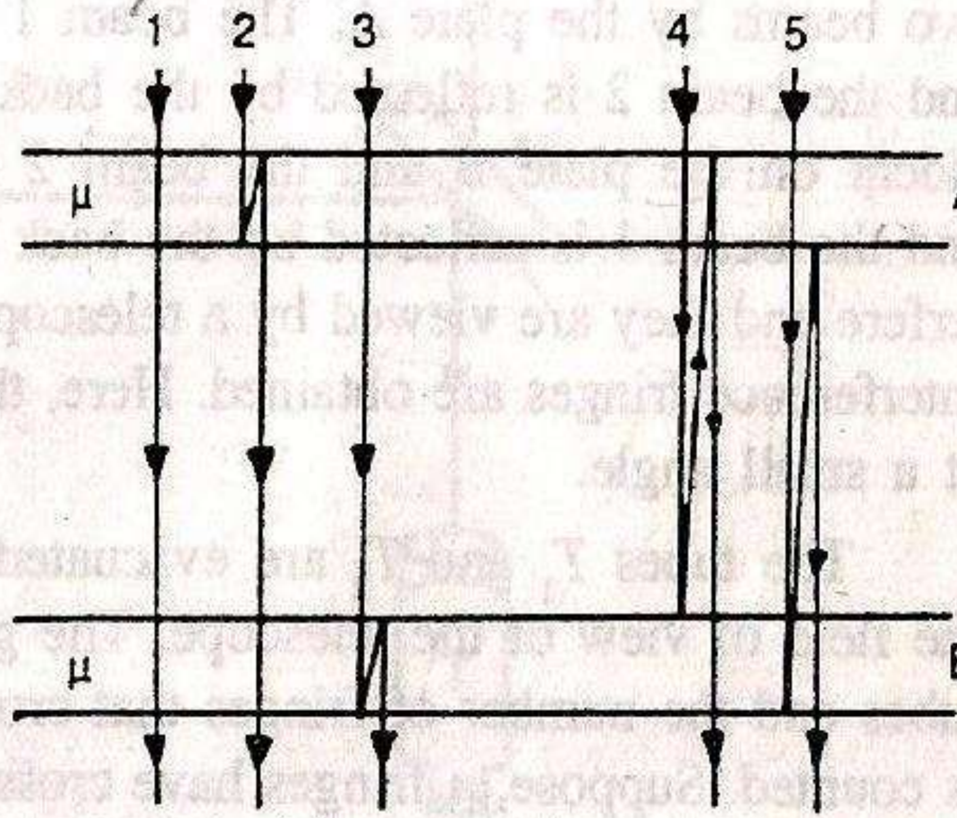


Fig. 8.46

Consider two thick plates *A* and *B* of thickness *t* and refractive index μ. Suppose, the two plates are parallel and several paths are traversed by a ray of light

(Fig. 8.46). The paths traversed by 2 and 3 are equal and if the plates are slightly inclined, a small path difference between 2 and 3 is introduced and interference fringes are observed., Similarly paths traversed by 4 and 5 are equal. If the plates *A* and *B* are slightly inclined a small path difference is introduced and interference fringes are observed. These fringes are known as Brewster's fringes.

8.32 MICHELSON INTERFEROMETER

Michelson interferometer consists of two highly polished mirrors M_1 and M_2 and two plane glass plates A and C parallel to each other. The rear side of the glass plate A is half silvered so that light coming from the source S is equally reflected and transmitted by it. Light from a monochromatic source S after passing through the lens L, falls on the plate A. The lens L makes the beam parallel. The plate A is inclined at an angle of 45°. One half of the energy of the incident beam is reflected by the plate A towards the mirror M_1 and the other half is transmitted towards the mirror M_2. These two beams (reflected and transmitted) travel along two mutually perpendicular paths and are reflected back by the mirror M_1 and M_2. These two beams return to the plate A. The beam reflected back by M_1 is transmitted through the glass plate A and the beam reflected back by M_2 is reflected by the glass plate A towards the eye (Fig. 8.37). The beam going towards the mirror M_1 and reflected back, has to pass twice through the glass plate A. Therefore, to compensate for the path, the plate C is used between the mirror M_2 and A. The light beam going towards the mirror M_2 and reflected back towards A also passes twice through the compensation plate C. Therefore, the paths of the two rays in glass are the same. The mirror M_1 is fixed on a carriage and can be moved with the help of the handle H. The distance through which the mirror M_1 is

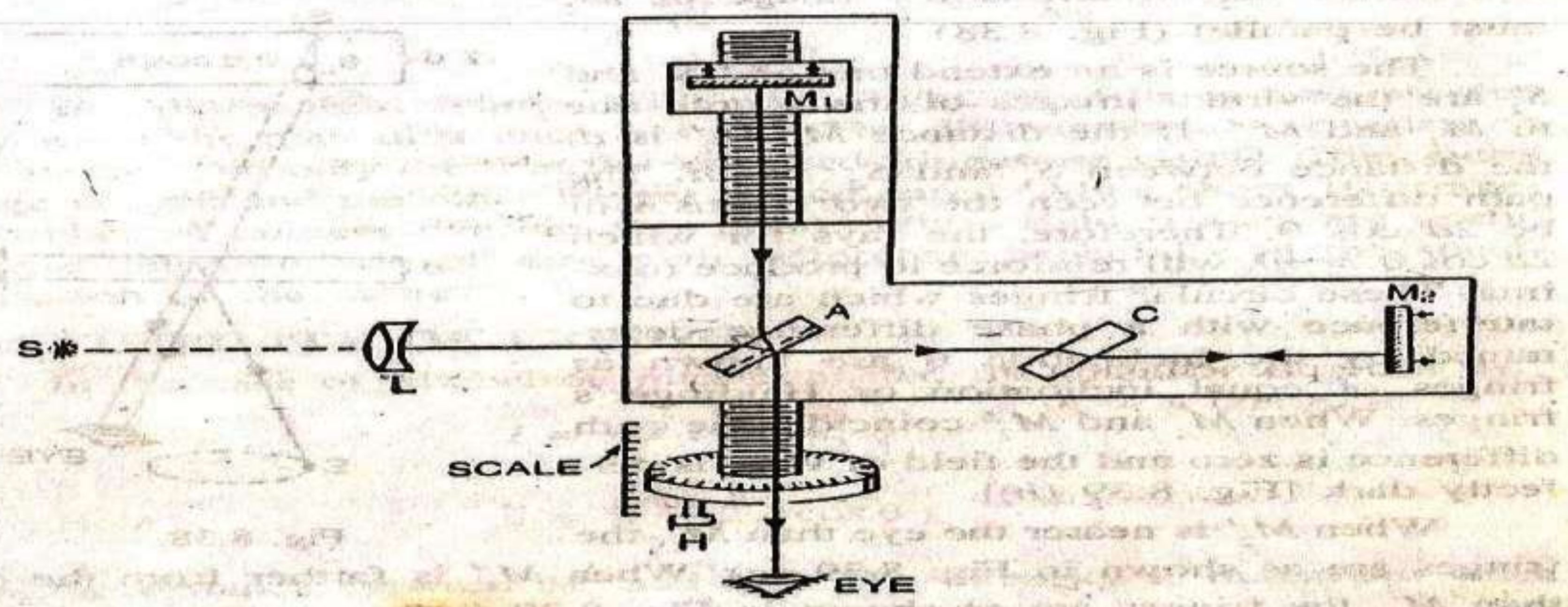


Fig. 8.37

moved can be read on the scale. The planes of the mirrors M_1 and M_2 can be made perfectly perpendicular with the help of the fine screws attached to them. The compensating plate is a necessity for white light fringes but can be dispensed with, while using monochromatic light.

If the mirrors M_1 and M_2 are perfectly perpendicular, the observer's eye will see the images of the mirrors M_1 and M_2 through A. There will be an air film between the two images and the distance can be varied with the help of the handle H. The fringes will be perfectly circular. If the path travelled by the two rays is exactly the same, the field of view will be completely dark. If the two images of M_1 and M_2 are inclined (the mirrors M_1 and M_2 not perfectly perpendicular) the enclosed air film will be wedge shaped and straight line fringes will be observed. When the mirror M_1 is moved away or towards the glass plate A with the help of the handle H, the fringes cross the centre of the field of view of the observer's eye. If M_1 is moved through a distance $\lambda/2$, one fringe will cross the field of view and will move to the position previously occupied by the next fringe.

8.33 TYPES OF FRINGES

(*i*) **Circular fringes.** Circular fringes are produced with monochromatic light in a Michelson interferometer. Here, the mirror M_1 and the virtual mirror M_2' which is the image of M_2 must be parallel (Fig. 8.38).

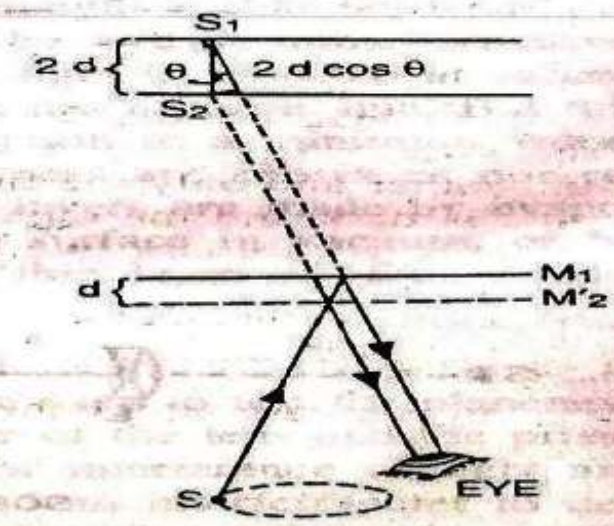


Fig. 8.38.

The source is an extend one and S_1 and S_2 are the virtual images of the source due to M_1 and M_2'. If the distance $M_1 M_2'$ is d, the distance between S_1 and S_2 $= 2d$. The path difference between the two beams will be $2d \cos \theta$. Therefore, the rays for which $2d \cos \theta = n\lambda$ will reinforce to produce maxima. These circular fringes which are due to interference with a phase difference determined by the inclination θ are known as fringes of equal inclination or Haidinger's fringes. When M_1 and M_2' coincide, the path difference is zero and the field of view is perfectly dark [Fig. 8.39 (*b*)].

When M_2' is nearer the eye than M_1, the fringes are as shown in Fig. 8.39 (*a*). When M_1' is farther from the eye than M_1, the fringes are as shown in Fig. 8.39 (*c*).

(*ii*) **Localized fringes.** When the mirror M_1 and the virtual mirror M_2' (image of M_2) are inclined, the air film enclosed is wedge- shaped and straight line fringes are observed. The shape of the fringes observed for various values of the path difference are shown in Fig. 8.40. The fringes are perfectly straight when M_1 actually intersects M_2' in the middle [Fig. 8.40 (*ii*)]. In the other positions, the shape of the fringes is as shown in Fig. 8.40 (*i*) and (*iii*). They are curved and are always convex towards the thin edge of the wedge. This type of fringes are not observed for large path differences.

Fig. 8.39

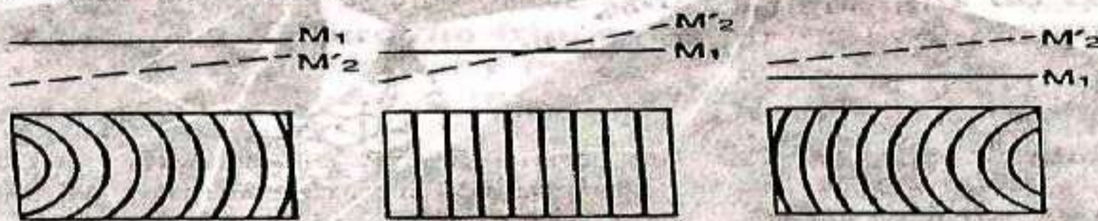


Fig. 8.40

(*iii*) **White light fringes.** With white light, the fringes are observed only when the path difference is small. The different colours overlap on one another and only the first few coloured fringes are visible. The central fringe is dark and the other fringes are coloured. After about 10 fringes a number of colours overlap at a point. White light fringes are useful for the determination of zero path difference, especially in the standardisation of the metre.

8.34 VISIBILITY OF FRINGES

In the case of Michelson interferometer, the intensity is given by

$$I = 4a^2 \cos^2\left(\frac{\delta}{2}\right)$$

Here $$\delta = \frac{2\pi}{\lambda}(2d\cos\theta)$$

d is the distance between M_1 and M_2'. The intensity is maximum when δ is an integral multiple of 2π. The intensity is zero when δ is an odd multiple of π. When a monochromatic source of light is used, the minimum intensity of the fringes is zero. The visibility of fringes in the case of a Michelson interferometer is

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

For monochromatic light,

$$I_{min} = 0$$

$$\therefore \quad V = 1$$

However, if the source of light is not strictly monochromatic, but contains two nearby wavelengths, the condition for maximum intensity for both the wavelengths is satisfied only for particular values of path difference ($2d \cos \theta$).

As the value of d is altered, the two wavelengths do coincide over a considerable range and here the fringe visibility is maximum. For values of d other than maximum intensity positions for both the wavelengths, the two fringe patterns will be complimentary, provided the intensities for both the wavelengths are equal. If intensities are not equal, the minimum visibility will not be zero. The minimum visibility will be

$$V_{min} = \frac{a_1^2 - a_2^2}{a_1^2 + a_2^2}$$

Here a_1 and a_2 are the amplitudes.

Hence the source will be perfectly monochromatic if visibility is maximum and constant for different values of $2d \cos \theta$. If the visibility changes with the change of $2d \cos \theta$, the source is not strictly monochromatic.

8.35. APPLICATIONS OF MICHELSON INTERFEROMETER

Michelson interferometer can be used to determine (*i*) the wavelength of a given monochromatic source of light, (*ii*) the difference between the two neighbouring wavelengths or resolution of the spectral lines, (*iii*) refractive index and thickness of various thin transparent materials and (*iv*) for the measurement of the standard metre in terms of the wavelength of light.

8.36. DETERMINATION OF THE WAVELENGTH OF MONOCHROMATIC LIGHT

The mirrors M_1 and M_2 are adjusted so that circular fringes are visible in the field of view (Fig. 8.39). If M_1 and M_2 are equidistant from the glass plate A, the field of view will be perfectly dark. The mirror M_2 is kept fixed and the mirror M_1 is moved with the help of the handle of the micrometer screw and the number of fringes that cross the field of view is counted. Suppose for the monochromatic light of wavelength λ, the distance through which the mirror is moved $= d$ and the number of fringes that cross the centre of the field of view $= n$. Then, $d = \frac{n\lambda}{2}$, because for one fringe shift, the mirror moves through a distance equal to half the wavelength. Hence λ can be determined.

8.37 DETERMINATION OF THE DIFFERENCE IN WAVELENGTH BETWEEN TWO NEIGHBOURING SPECTRAL LINES (RESOLUTION OF THE SPECTRAL LINES)

There are two spectral lines D_1 and D_2 of sodium light. They are very near to each other and the difference in their wavelengths is small. Suppose, the wavelength of D_1 line is λ_1 and the wavelength of D_2 line is λ_2. Also $\lambda_1 \neq \lambda_2$. Each spectral line will give rise to its fringes in a Michelson interferometer. By adjusting the position of the mirror M_1 of the Michelson interferometer, the position is found when the fringes are very bright. In this position, the bright fringes due to D_1 coincides with the bright fringes due to D_2. When the mirror M_1 is moved, the two sets of fringes get out of step because their wavelengths are different. When the mirror M_1 is moved through a certain distance, the bright fringes due to one set will be seen in this case. Again by moving the mirror M_1, a position is reached when a bright fringe of one set falls on the bright fringe of the other and the fringes are again distinct. This is possible when the n th order of the longer wavelength coincides with the $(n + 1)$ th order of the shorter wavelength.

Let n_1 and n_2 be the changes in the order at the centre of the field of view, when the mirror M_1 is displaced through a distance d between two consecutive positions of maximum distinctness of the fringes.

$$\therefore \quad 2d = n_1\lambda_1 = n_2\lambda_2$$

If λ_1 is greater than λ_2

$$n_2 = n_1 + 1$$

$$\therefore \quad 2d = n_1\lambda_1 = (n_1 + 1)\lambda_2 \qquad ...(i)$$

$$n_1\lambda_1 = (n_1 + 1)\lambda_2$$

$$\therefore \quad n_1 = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

Substituting the value of n_1 in (i)

$$2d = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2}$$

or

$$\lambda_1 - \lambda_2 = \frac{\lambda_1\lambda_2}{2d} \qquad ...(ii)$$

Taking λ as the mean of λ_1 and λ_2

$$\therefore \quad \Delta\lambda = \lambda_1 - \lambda_2 = \frac{\lambda^2}{2d} \qquad ...(iii)$$

Hence the difference in wavelength $\lambda_1 - \lambda_2$ can be calculated. In actual practice, readings for ten successive positions of maximum distinctness are taken and the mean value of d is calculated.

Also, wave number

$$\bar{\nu}_1 = \frac{1}{\lambda_1} \text{ and } \bar{\nu}_2 = \frac{1}{\lambda_2}$$

From equation (ii),

$$\lambda_1 - \lambda_2 = \frac{\lambda_1\lambda_2}{2d}$$

or
$$\frac{1}{\lambda_2}-\frac{1}{\lambda_1}=\frac{1}{2d}$$

or
$$\bar{v}_2-\bar{v}_1=\frac{1}{2d} \qquad \text{...}(i)$$

This equation represents the difference in the wave number of the two spectral lines.

8.38 DETERMINATION OF REFRACTIVE INDEX OF THIN TRANSPARENT PLATES

When a thin transparent material (mica or cellophane) is introduced in the path of the beam going towards M_1, a path difference $2(\mu - 1)t$ is introduced between the two interfering beams. With monochromatic light, this path difference introduces a displacement in the fringe system. Suppose N fringes have crossed the centre of the field of view. But experimentally it is not possible to count this number N.

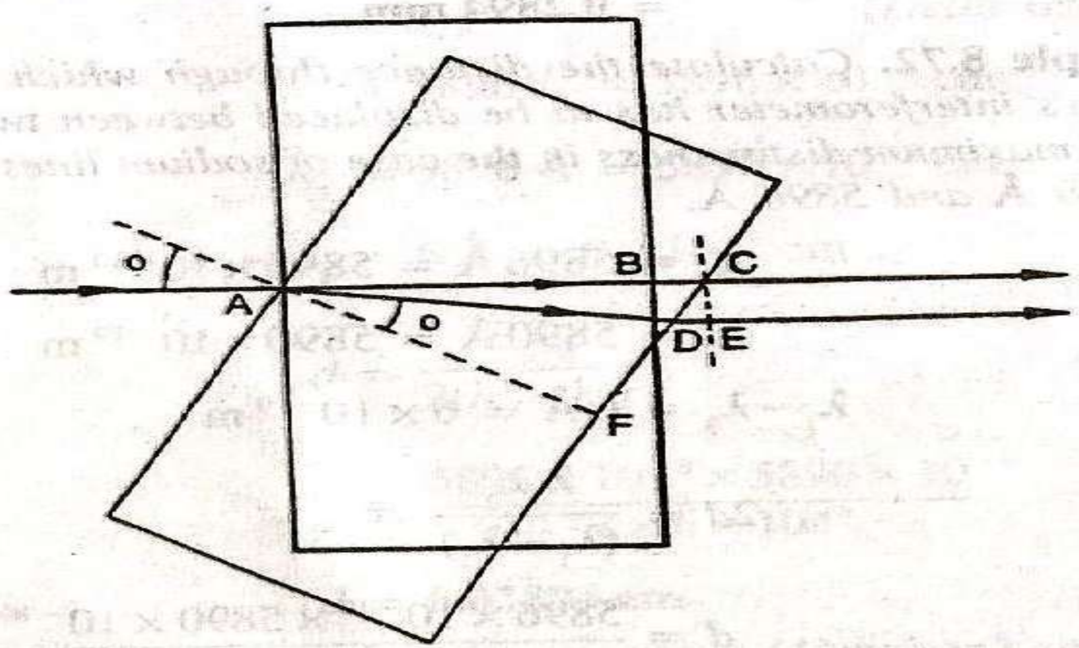


Fig. 8.41

The following method is used to count the number of fringes that cross the field of view.

(1) The given transparent plate is introduced in the path of the beam going towards M_1. The centre of the field of view is observed.

(2) The plate is slowly rotated and the number of fringes that cross the field of view is counted. Suppose for an angle of rotation ϕ, the number of fringes that cross the field of view is n.

In Fig. 8.41, the plate of thickness t, and refractive index μ has been rotated through an angle ϕ. The optical path for ABC is $\mu t + BC$ and for ADE it is $\mu AD + DE$.

The increase in optical path for n fringes that cross the field view is given by

$$2\,[\,\mu AD + DE - \mu t - BC] = n\lambda \qquad ...(i)$$

Here $\quad AD = \dfrac{t}{\cos\phi'}\,; \quad DE = DC\sin\phi$

$$DC = FC - FD = t\tan\phi - t\tan\phi'$$

$$BC = \frac{t}{\cos\phi} - t$$

Substituting these values in equation (i)

$$\frac{\mu t}{\cos\phi'} + t\sin\phi\,(\tan\phi - \tan\phi') - \mu t - t\left(\frac{1}{\cos\phi} - 1\right) = \frac{n\lambda}{2} \qquad ...(ii)$$

But $\quad \mu = \dfrac{\sin\phi}{\sin\phi'}$

Simplifying equation (ii)

$$\frac{\mu}{\cos\phi'} + \sin\phi\,(\tan\phi - \tan\phi') - \frac{1}{\cos\phi} = \frac{n\lambda}{2t} - 1 + \mu$$

$$\frac{\mu}{\cos\phi'} + \frac{\sin^2\phi}{\cos\phi} - \frac{\sin\phi\sin\phi'}{\cos\phi'} - \frac{1}{\cos\phi} = \frac{n\lambda}{2t} - 1 + \mu$$

$$\frac{\mu}{\cos\phi'} - \frac{\sin^2\phi}{\mu\cos\phi'} + \frac{\sin^2\phi}{\cos\phi} - \frac{1}{\cos\phi} = \frac{n\lambda}{2t} - 1 + \mu$$

$$\frac{1}{\mu\cos\phi'}\,[\mu^2 - \sin^2\phi] - \frac{1}{\cos\phi}\,(1 - \sin^2\phi) = \frac{n\lambda}{2t} - 1 + \mu$$

But $\quad \mu\cos\phi' = \sqrt{\mu^2 - \sin^2\phi}$

$$\therefore\ (\sqrt{\mu^2 - \sin^2\phi}) - \cos\phi = \frac{n\lambda}{2t} - 1 + \mu$$

$$\mu = \frac{(2t - n\lambda)(1 - \cos\phi) + (n^2\lambda^2/4t)}{2t(1 - \cos\phi) - n\lambda} \qquad ...(iii)$$

Hence μ can be determined from relation (*iii*). The term $n^2\lambda^2 / 4t$ can be neglected as it very small.

8.39 DETERMINATION OF THE REFRACTIVE INDEX OF GASES

When a tube containing a gas is introduced in the path of the beam going towards M_1, a path difference $= 2(\mu - 1) l$ is introduced between the two interfering beams. Here, μ is the refractive index of the gas and l is the length of the tube. If n fringes cross the centre of the field of view, $2(\mu - 1) l = n\lambda$. Knowing l, n and λ, μ can be calculated. If μ, n and λ are known, l can be calculated.

In the path of the rays going towards M_1 , a tube containing air at atmospheric pressure is introduced and the fringes are obtained in the centre of the field of view. In that case, refractive index of air at various pressures can be determined. Let the length of the tube be l and let it contain air at atmospheric pressure. The tube is completely evacuated and n fringes cross the centre of the field of view. The path difference introduced between the two interfering beams

$$= 2(\mu - 1) l$$

$$\therefore \quad 2(\mu - 1) l = n\lambda$$

or

$$\mu = \frac{n\lambda}{2l} + 1$$

8.45 FABRY-PEROT INTERFEROMETER

A Fabry- Perot interferometer consists of two plane parallel glass plates *A* and *B*. With this interferometer fringes of constant inclination are obtained by transmitted light after multiple reflection between the glass plates (Fig. 8.50)

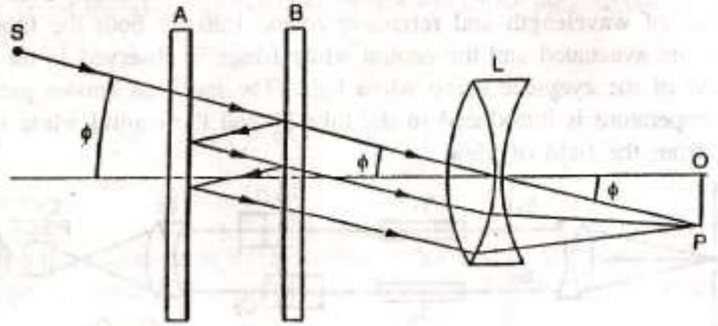


Fig. 8.50

The inside surfaces of the two plates are silvered. Here the multiple reflections take place in the air film between the plates *A* and *B*. In Fig. 8.50 a ray of light from a point *S* on an extended source, after multiple reflection is brought to focus at the point *P* by the lens *L*. The condition for maximum intensity by transmitted light in a plane parallel air film is given by

$$2\mu t \cos\phi = n\lambda$$

For an air film $\mu = 1$

$$\therefore \quad 2t\cos\phi = n\lambda \qquad \text{...(i)}$$

where $n = 0, 1, 2, 3$......etc.

The interference pattern consists of concentric rings with *O* as centre. Each ring corresponds to a particular value of θ. The radius of the ring is *OP*. Fringes of constant inclination are called **Haidinger Fringes**.

In this interferometer, the two plates are invariably kept parallel. One of the plates is fixed and the other plate is moved with a rack and pinion arrangement. If the distance t between the two plates is decreased, the value of ϕ decreases for a given value of λ and n. It means with decrease in t, the rings shrink and disappear at the centre. Whenever t is decreased by $\frac{\lambda}{2}$, one ring disappears at the centre. Also the order of the rings decreases from centre outwards. The ring pattern of a Fabry-Perot interferometer is extremely sharp in comparison to the ring pattern obtained with a Michelson interferometer. Hence Fabry-Perot interferometer is very useful for resolving very small wavelength differences. The fringes may be obtained with plate separation up to 10 cm.

Interferometers based on multiple reflections will give fringes which are very sharp in comparison to those obtained with two coherent sources. In a Fabry-Perot interferometer, the sharpness of the fringes depends upon the reflection coefficient of the silvered surfaces. Suppose, the transmitted amplitudes are C, Cr^2, Cr^4, Cr^6...etc. Here r is the reflection coefficient and C is the product of the original amplitude and the transmission co-efficient at the two plates.

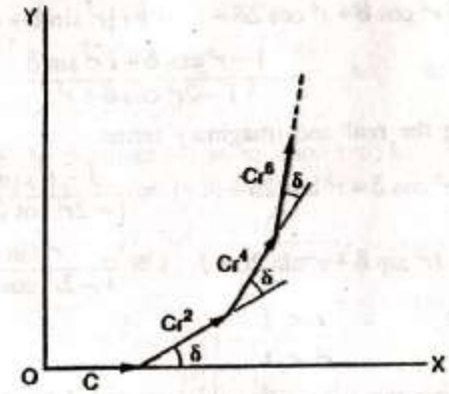


Fig. 8.51

The transmitted beams have a constant phase difference δ where

$$\delta = \left(\frac{4\pi t}{\lambda}\right)\cos\phi$$

Applying the law of polygon of vectors (Fig. 8.51), the intensity I is proportional to the square of the resultant amplitude.

$$I \propto (C + Cr^2\cos\delta + Cr^4\cos 2\delta + \ldots)^2 + (Cr^2\sin\delta + Cr^4\sin 2\delta + \ldots)^2$$

$$I \propto C^2\,[(1 + r^2\cos\delta + r^4\cos 2\delta + \ldots)^2 + (r^2\sin\delta + r^4\sin 2\delta + \ldots)^2] \qquad \text{...(ii)}$$

This can be evaluated as follows :

Mathematically,

$$1 + r^2 e^{i\delta} + r^4 e^{i2\delta} + r^6 e^{i3\delta} + \ldots = \frac{1}{1 - r^2 e^{i\delta}}$$

Here $\quad i = \sqrt{-1}$

Multiply the numerator and denominator of the *R.H.S.* by $1 - r^2 e^{-i\delta}$

$$\therefore \quad 1 + r^2 e^{i\delta} + r^4 e^{i2\delta} + r^6 e^{i3\delta} + \ldots = \frac{1 - r^2 e^{-i\delta}}{1 - r^2(e^{i\delta} + e^{-i\delta}) + r^4} \qquad \text{...(iii)}$$

Also $\quad e^{\pm i\delta} = \cos\delta \pm i\sin\delta$

Substituting its value in equation (*iii*)

$$1 + r^2(\cos\delta + i\sin\delta) + r^4(\cos 2\delta + i\sin 2\delta) + \ldots$$
$$= \frac{1 - r^2(\cos\delta - i\sin\delta)}{1 - r^2[\cos\delta + i\sin\delta + \cos\delta - i\sin\delta] + r^4}$$

Separating the real and imaginary term,

$$(1 + r^2\cos\delta + r^4\cos 2\delta + \ldots) + i\,[r^2\sin\delta + r^4\sin 2\delta + \ldots]$$
$$= \frac{1 - r^2\cos\delta + i\,r^2\sin\delta}{1 - 2r^2\cos\delta + r^4} \qquad \ldots(iv)$$

Equating the real and imaginary terms

$$(1 + r^2\cos\delta + r^4\cos 2\delta + \ldots) = \frac{1 - r^2\cos\delta}{1 - 2r^2\cos\delta + r^4} \qquad \ldots(v)$$

and

$$(r^2\sin\delta + r^4\sin 2\delta + \ldots) = \frac{r^2\sin\delta}{1 - 2r^2\cos\delta + r^4} \qquad \ldots(vi)$$

As $\quad r < 1$

$\therefore \quad r^2 < 1$

Substituting the values in equations (*v*) and (*vi*) in equation (*ii*)

$$I \propto C^2\left[\frac{(1 - r^2\cos\delta)^2 + (r^2\sin\delta)^2}{(1 - 2r^2\cos\delta + r^4)^2}\right]$$

$$I \propto C^2\left[\frac{1 - 2r^2\cos\delta + r^4}{(1 - 2r^2\cos\delta + r^4)^2}\right]$$

$$I \propto \left(\frac{C^2}{1 - 2r^2\cos\delta + r^4}\right)$$

$$I \propto \frac{C^2}{\left(1 - r^2\right)^2 + 4r^2\sin^2\left(\frac{\delta}{2}\right)}$$

$$I = k\left[\frac{C^2}{(1 - r^2)^2 + 4r^2\sin^2\left(\frac{\delta}{2}\right)}\right] \qquad \ldots(vii)$$

Special Cases :

(1) When $\quad \delta = 0,\ 2\pi,\ 4\pi \ldots$etc.

$$\frac{\delta}{2} = 0,\ \pi,\ 2\pi \ldots \text{etc. and } \sin\frac{\delta}{2} = 0$$

In this case, the denominator in equation (*vii*) will have minimum value. Hence the intensity of the fringes will be maximum

$$I_{max} \propto \frac{C^2}{(1-r^2)^2} \qquad \text{...(viii)}$$

$$I_{max} = k\left[\frac{C^2}{(1-r^2)^2}\right]$$

(2) When $\delta = \pi, 3\pi, 5\pi, \ldots$ etc.

$$\frac{\delta}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \text{ and } \sin^2\left(\frac{\delta}{2}\right) = 1$$

In this case, the denominator in equation (*vii*) has a maximum value. Hence, intensity of the fringes will be minimum

$$I_{min} \propto \frac{C^2}{(1-r^2)^2+4r^2}$$

$$I_{min} \propto \frac{C^2}{(1+r^2)^2} \qquad \text{...(ix)}$$

$$I_{min} = k\left[\frac{C^2}{(1+r^2)^2}\right]$$

8.50 INTERFERENCE FILTER

An interference filter is based on the principle of Fabry-Perot interferometer. It consists of an optical system that will transmit nearly a monochromatic beam of light (covering a small range of 50 Å).

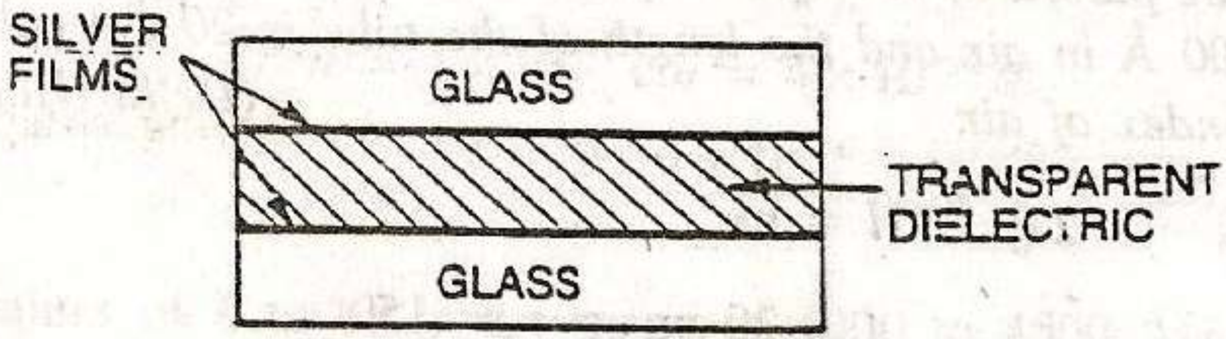


Fig. 8.56

An interference filter consists of a thin transparent dielectric e.g. magnesium fluoride. There are two glass plates on whose surfaces semi transparent silver films are deposited by evaporation method. The dielectric is placed between the two glass plates (Fig. 8.56).

When a beam of light is incident normally on the filter, multiple reflections take place within the film. The interference maxima for the transmitted beam will be governed by

$$2\mu t = n\lambda$$

Here μ is th refractive index of the dielectric and t is its thickness, and n is a whole number. If $\mu t = \lambda$, n will be equal to 2. For the value of $n = 1$, the maximum occurs for a wavelength of 2λ. Here λ and 2λ represent a wide separation in the visible region.

In the case of an interference filter, when the thickness of the dielectric is reduced, the transmitted wavelengths are more widely spaced. For an optical thickness (μt) of the dielectric film of 5000 Å, the transmitted wavelengths for $n = 1, 2, 3$ etc. are 10,000 Å, 5000 Å, 3333 Å. These three wavelengths are widely spaced. Only 5000 Å is in the visible region. If there are two maxima in the visible region one of them can be eliminated by using a coloured glass filter. This may be the protecting glass of the dielectric itself.

Interference filters are better as compared to the coloured glass filters because in the case of interference filters light is not absorbed and hence there is no overheating. Interference filters are used in spectroscopic work for studying the spectra in a narrow range of wavelengths.

8.52 STATIONARY WAVES IN LIGHT

Similar to stationary waves in sound, due to interference of incident and reflected sound waves in strings and air columns, stationary waves are produced in light. This gives another evidence of the wave nature of light. This phenomenon has been used in colour photography. when a light wave is travelling in the direction of the x axis and is reflected by a denser medium, a phase change of π occurs between the incident and the reflected light. For the incident wave,

$$y_1 = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right) \qquad ...(i)$$

For the reflected wave

$$y_2 = -a \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda} \right) \qquad ...(ii)$$

The resultant displacement y is given by

$$y = y_1 + y_2 = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) - a \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$$

$$y = -2a \sin \frac{2\pi x}{\lambda} \cos \frac{2\pi t}{T} \qquad \ldots(iii)$$

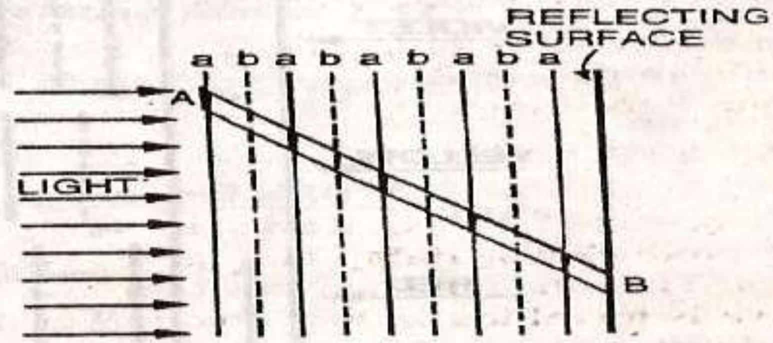
Here $2a \sin \frac{2\pi x}{\lambda}$ represents the amplitude of the resultant wave.

For $x = 0, \frac{\lambda}{2}, \ldots\ldots, \frac{n\lambda}{2}$, the amplitude is zero. These are the positions of nodes.

For $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} \ldots\ldots\ldots \frac{(2n+1)\lambda}{4}$, the amplitude is maximum. These are the positions of antinodes. These waves showing alternate formation of nodes and antinodes due to the interference of the incident and the reflected waves are called stationary waves in light.

Wiener Experiment. Wiener in 1890 demonstrated the stationary waves formed due to interference of the incident and the reflected light.

In his experiment, Wieners took a highly polished mirror as a reflecting surface. An extremely thin film of photographic emulsion was inclined at a small angle to the reflecting surface (Fig. 8.57). When the film was exposed to light and developed, the film was found to be crossed by a series of dark lines at the positions shown in Fig. 8.57. These are the positions of the antinodal planes. However, no blackening was observed at the nodal planes. This experiment shows clearly the formation of stationary waves of light due to interference of the incident and reflected light. In his experiment, Wiener also showed that it is the electric vector (E) perpendicular to the plane of incidence that is responsible for the production of these waves. Moreover, the light must be plane polarized.



a, a – ANTINODAL PLANES
b, b — NODAL PLANES

Fig. 8.57

In 1932, Fry and Ives repeated Wiener's experiment using the photoelectric effect in a thin metal film on a wedge of quartz in place of the photographic emulsion.

8.53 LIPPMANN'S COLOUR PHOTOGRAPHY

Lippmann in 1881 made use of the phenomenon of stationary waves in light in colour photography.

A photographic plate having an extremely fine grain ...

ered with mercury on
and mercury (Fig. 8.5
photographic plate wi
Let light of various w
plate as shown in Fig
Therefore the distance
light will be less. The
distance between two
Each colour forms its
such an exposed plate i
located at the antinode
layers for one particul

of the same wavelength (coherent) reflected from c which the layers were formed, are in phase with each reinforce and the reflected light of that particular tensity. Therefore a particular set of layers corres wavelength. The other colours reflected by the laye difference is not one wavelength (non coherent) will intensity will be much less. If n waves of amplitude sultant intensity is $(nA)^2$ in the case of coherent wav in the case of non-coherent waves. In the case of no resultant intensity is nA^2. To conclude, a particular se the path difference between reflected light is λ will r ticular colour is strongly reflected by which these l exposed.

The colour photographs due to Lippmann pr bright. But no prints can be taken from it and the p plates is tedious.

Note. Lippmann process is not used commercial ess is based on three colour process. The two methods and (*ii*) subtractive.

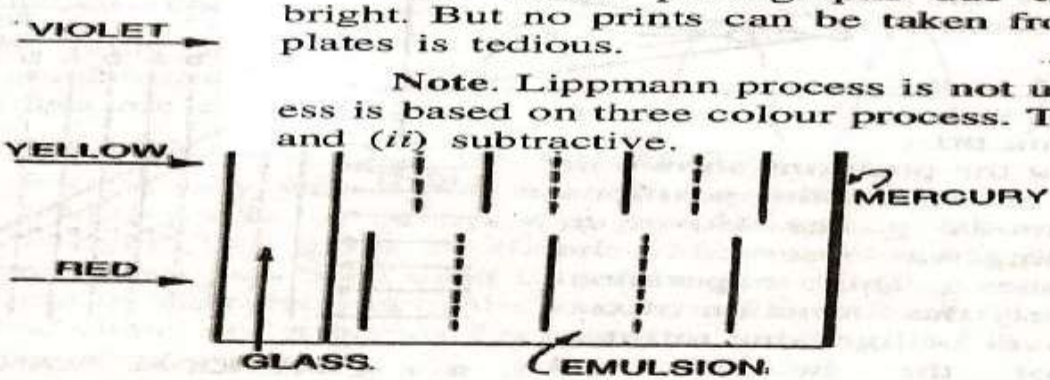


Fig. 8.58

When such a developed plate is illuminated with white light, each layer will reflect strongly only that particular colour by which it was originally formed. Here, each layer will reflect only a small amount of light. As white light passes through each layer, is gets reflected. All radiations of the same wavelength (coherent) reflected from different layers due to which the layers were formed, are in phase with each other. Therefore they reinforce and the reflected light of that particular colour is of large intensity. Therefore a particular set of layers corresponds to a particular wavelength. The other colours reflected by the layers for which the path difference is not one wavelength (non coherent) will not reinforce and their intensity will be much less. If n waves of amplitude A combine, their resultant intensity is $(nA)^2$ in the case of coherent waves but it is much less in the case of non-coherent waves. In the case of non-coherent waves the resultant intensity is nA^2. To conclude, a particular set of layers for which the path difference between reflected light is λ will reinforce and that particular colour is strongly reflected by which these layers were originally exposed.

The colour photographs due to Lippmann process are extremely bright. But no prints can be taken from it and the process to prepare the plates is tedious.

Note. Lippmann process is not used commercially. The modern process is based on three colour process. The two methods used are (*i*) additive and (*ii*) subtractive.

8.54 HOLOGRAPHY

In the case of ordinary photographs, it is possible only to obtain the view in a particular plane. The camera lens can be focussed only in a particular plane and the details of the field nearer and farther than the focussed plane will not be recorded. All the other planes are out of focus. The main reason for this is that the photograph records only the intensity distribution in a particular plane.

If, on the other hand, it is possible to record the *amplitude* and the *phase distribution* in any plane between the object and the observer, it helps to obtain the complete field of view as originally observed.

This type of recording is done in holography and holographic studies make it possible to have complete study of the field of view at any other time.

Gabor (1948) has introduced for the first time the holographic method of recording and retrieving the image.

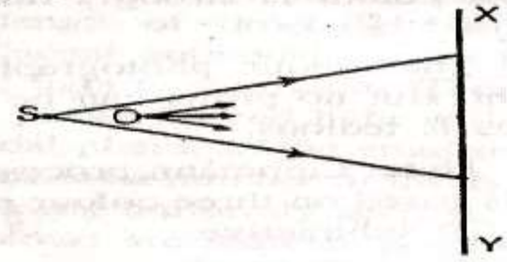


Fig. 8.59

S is a point source of light and *O* is a small object. On *XY*, the secondary wavelets from *O* superimpose on the strong primary waves from *S*. As the primary wave is uniform and more intense than the secondary wave, the variation in intensity across *XY* is dependent on the variation in phase across it. It is not determined by the variation in intensity across the secondary wave. In other words, the presence of strong coherent background helps to record information about the phase of the diffracted light. This technique was first introduced by Zernike. The pattern obtained on *XY* is called a hologram and its photograph is taken, keeping the time of exposure extremely small.

For reconstructing the field of view, the photographic plate is developed by reversal and if this developed plate is inserted at the place *XY* and only a source *S* is used, on looking through this plate towards *S*, the object will appear at the point *O*. In this way the original field of view is observed.

Holography has been used in holographic interferometry. Holography is also useful in the microscopic examination of certain kinds of specimens. If one desires to make a prolonged examination of a small specimen suspended in a medium, it is necessary to focus the microscope off and on due to the change of position of the specimen. This difficulty can be overcome by taking a short exposure holograph of the specimen. The reconstructed holographic image can be examined continuously by focussing the microscope.

Holography is also useful to provide a high capacity system for image storage and reexamination.

10

POLARIZATION

10.1 INTRODUCTION

Experiments on interference and diffraction have shown that light is a form of wave motion. These effects do not tell us about the type of wave motion *i.e.*, whether the light waves are longitudinal or transverse, or whether the vibrations are linear, circular or torsional. The phenomenon of polarization has helped to establish beyond doubt that light waves are transverse waves.

10.2 POLARIZATION OF TRANSVERSE WAVES

Let a rope *AB* be passed through two parallel slits S_1 and S_2. The rope is attached to a fixed point at *B* [Fig. 10.1(*a*)]. Hold the end *A* and

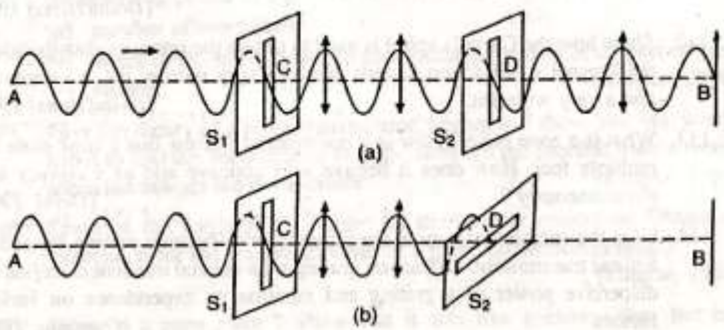


Fig. 10.1

move the rope up and down perpendicular to *AB*. A wave emerges along *CD* and it is due to transverse vibrations parallel to the slit S_1. The slit S_2 allows the wave to pass through it when it is parallel to S_1. It is observed that the slit S_2 does not allow the wave to pass through it when it is at right angles to the slit S_1 [Fig. 10.1(*b*)].

If the end *A* is moved in a circular manner, the rope will show circular motion up to the slit S_1. Beyond S_1, it will show only linear vibrations parallel to the slit S_1, because the slit S_1 will stop the other components. If S_1 and S_2 are at right angles to each other the rope will not show any vibration beyond S_2.

If longitudinal waves are set up by moving the rope forward and backward along the string, the waves will pass through S_1 and S_2 irrespective of their position.

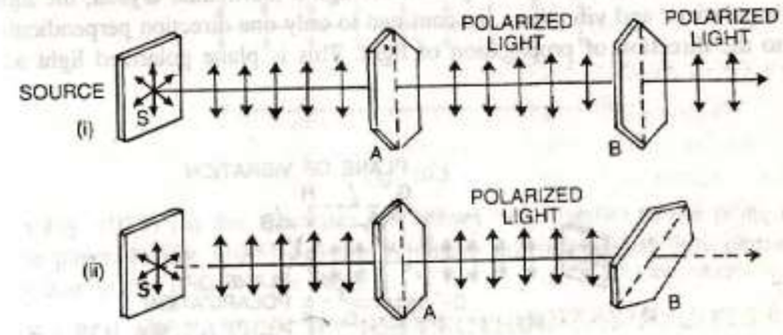


Fig. 10.2

A similar phenomenon has been observed in light when it passes through a tourmaline crystal.

Let light from a source *S* fall on a tourmaline crystal *A* which is cut parallel to its axis (Fig. 10.2). The crystal *A* will act as the slit S_1. The light is slightly coloured due to the natural colour of the crystal. On rotating the crystal *A*, no remarkable change is noticed. Now place the crystal *B* parallel to *A*.

(1) Rotate both the crystals together so that their axes are always parallel. No change is observed in the light coming out of *B* [Fig. **10.2** (*i*)].

(2) Keep the crystal *A* fixed and rotate the crystal *B*. The light transmitted through *B* becomes dimmer and dimmer. When *B is at right angles to A*, no light emerges out of *B* [Fig. 10.2 (*ii*)].

If the crystal *B* is further rotated, the intensity of light coming out of it gradually increases and is maximum again when the two crystals are parallel.

This experiment shows conclusively that light is not propagated as longitudinal or compressional waves. If we consider the propagation of light as a longitudinal wave motion then no extinction of light should occur when the crystal *B* is rotated.

It is clear that after passing through the crystal *A*, the light waves vibrate only in one direction. Therefore light coming out of the crystal *A* is said to be **polarized** because it has acquired the property of **one sidedness** with regard to the direction of the rays.

This experiment proves that light waves are transverse waves, otherwise light coming out of *B* could never be extinguished by simply rotating the crystal *B*.

10.3 PLANE OF POLARIZATION

When ordinary light is passed through a tourmaline crystal, the light is polarized and vibrations are confined to only one direction perpendicular to the direction of propagation of light. This is plane polarized light and

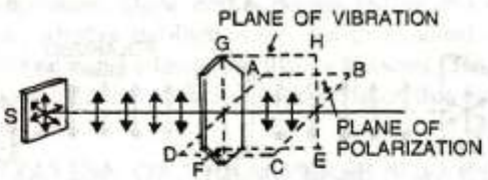


Fig. 10.3

it has acquired the property of one sidedness. The plane of polarization is that plane in which no vibrations occur. The plane *ABCD* in Fig. 10.3 is the plane of polarization. The vibrations occur at right angles to the plane of polarization and the plane in which vibrations occur is known as plane of vibration. The plane *EFGH* in Fig. 10.3 is the plane of vibration.

Ordinary light from a source has very large number of wavelengths. Moreover, the vibrations may be linear, circular or elliptical. From our idea of wave motion, circular or elliptical vibrations consist of two linear vibrations at right angles to each other and having a phase difference of $\frac{\pi}{2}$.

Therefore any vibration can be resolved into two component vibrations at right angles to each other. As light waves are transverse waves the vibrations can be resolved into two planes xx' and yy'

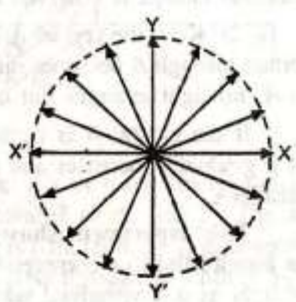


Fig. 10.4

at right angles to each other and also perpendicular to the direction of propagation of light (Fig. 10.4).

In Fig. 10.5(*i*), the vibrations of the particles are represented parallel (arrow heads) and perpendicular to the plane of the paper (dots).

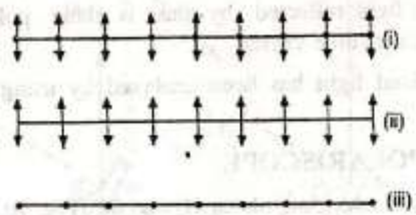


Fig. 10.5

In Fig. (10.3) (*ii*) the vibrations are shown only parallel to the plane of the paper. In Fig. (10.5) (*iii*) the vibrations are represented only perpendicular to the plane of the paper.

10.4 POLARIZATION BY REFLECTION

Polarization of light by reflection from the surface of glass was discovered by Malus in 1808. He found that polarized light is obtained when ordinary light is reflected by a plane sheet of glass. Consider the light incident along the path *AB* on the glass surface (Fig. 10.6). Light is

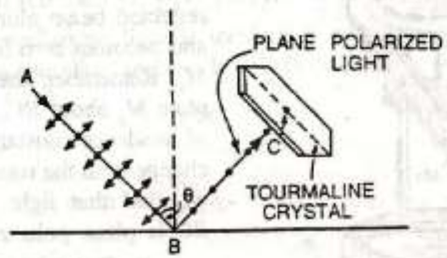


Fig. 10.6

reflected along *BC*. In the path of *BC*, place a tourmaline crystal and rotate it slowly. It will be observed that light is completely extinguished only at one particular angle of incidence. This angle of incidence is equal to 57.5° for a glass surface and is known as the polarizing angle. Similarly polarized light by reflection can be produced from water surface also.

The production of polarized light by glass is explained as follows. The vibrations of the incident light can be resolved into components parallel to the glass surface and perpendicular to the glass surface. Light due to the components parallel to the glass surface is reflected whereas light due to the components perpendicular to the glass surface is transmitted.

Thus, the light reflected by glass is plane polarized and can be detected by a tourmaline crystal.

The polarized light has been analysed by using another mirror by Biot.

10.5 BIOTS POLARISCOPE

It consists of two glass plates M_1 and M_2 (Fig. 10.7). The glass plates are painted black on their back surfaces so as to avoid any reflection and this also helps in absorbing refracted light. A beam of unpolarized light AB is incident at an angle of about 57.5° on the first glass surface at B and is reflected along BC (Fig. 10.8). This light is again reflected at 57.5° by the second glass plate M_2 placed parallel to the first. The glass plate M_1 is known as the polarizer and M_2 as the analyser.

Fig. 10.7

When the upper plate M_2 is rotated about BC, the intensity of the reflected beam along CD decreases and becomes zero for 90° rotation of M_2. Remember, the rotation of the plate M_2 about BC, keeps the angle of incidence constant and it does not change with the rotation of M_2. Thus we find that light travelling along BC is plane polarized.

When the mirror M_2 is rotated further it is found that the intensity of CD becomes maximum at 180°, minimum at 270° and again maximum at 360°.

The above experiment proves that when light is incident at an angle of 57.5° on a glass surface, the reflected light consists of waves in which

the displacements are confined to a certain direction at right angles to the ray and we get polarized light by reflection.

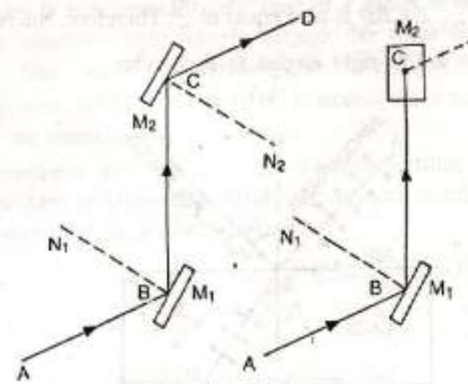


Fig. 10.8

10.6 BREWSTER'S LAW

In 1811, Brewster performed a number of experiments to study the polarization of light by reflection at the surfaces of different media.

He found that ordinary light is completely polarized in the plane of incidence when it gets reflected from a transparent medium at a particular angle known as the **angle of polarization**.

He was able to prove that the tangent of the angle of polarization is numerically equal to the refractive index of the medium. Moreover, the reflected and the refracted rays are perpendicular to each other.

Suppose, unpolarized light is incident at an angle equal to the polarizing angle on the glass surface. It is reflected along BC and refracted along BD (Fig. 10.9).

From Snell's law

$$\mu = \frac{\sin i}{\sin r} \quad ...(i)$$

From Brewster's law

$$\mu = \tan i = \frac{\sin i}{\cos i} \quad ...(ii)$$

Comparing (*i*) and (*ii*)

$$\cos i = \sin r = \cos\left(\frac{\pi}{2} - r\right)$$

$$\therefore \qquad i = \frac{\pi}{2} - r, \quad \text{or} \quad i + r = \frac{\pi}{2}$$

As $i + r = \frac{\pi}{2}$, $\angle CBD$ is also equal to $\frac{\pi}{2}$. Therefore, the reflected and the refracted rays are at right angles to each other.

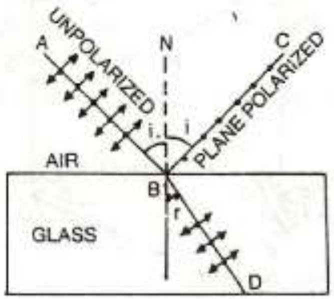


Fig. 10.9

From Brewster's law, it is clear that for crown glass of refractive index 1.52, the value of i is given by

$$i = \tan^{-1}(1.52) \quad \text{or} \quad i = 56.7^\circ$$

However, 57° is an approximate value for the polarizing angle for ordinary glass. For a refractive index of 1.7 the polarising angle is about 59.5° *i.e.*, the polarizing angle is not widely different for different glasses.

As the refractive index of a substance varies with the wavelength of the incident light, the polarizing angle will be different for light of different wavelengths. Therefore, polarization will be complete only for light of a particular wavelength at a time *i.e.*, for monochromatic light.

It is clear that the light vibrating in the plane of incidence is not reflected along BC [Fig. 10.9]. In the reflected beam the vibrations along BC cannot be observed, whereas vibrations at right angles to the plane of incidence can contribute for the resultant intensity. Thus, we get plane polarized light along BC. The refracted ray will have both the vibrations (*i*) in the plane of incidence and (*ii*) at right angles to the plane of incidence. But it is richer in vibrations in the plane of incidence. Hence it is partially plane-polarized.

10.7 BREWSTER WINDOW

One of the important applications of Brewster's law and Brewster's angle is in the design of a glass window that enables 100% transmission of light. Such a type of window is used in lasers and it is called a **Brewster window**.

When an ordinary beam of light is incident normally on a glass window, about 8% of light is lost by reflection on its two surfaces and about 92% intensity is transmitted. In the case of a gas laser filled with mirrors outside the windows, light travels through the window about a hundred times. In this way the intensity of the final beam is about 3×10^{-4} because $(0.92)^{100} \approx 3 \times 10^{-4}$. It means the transmitted beam has practically no intensity.

To overcome this difficulty, the window is tilted so that the light beam is incident at Brewster's angle. After about hundred transmissions, the final beam will be plane polarized.

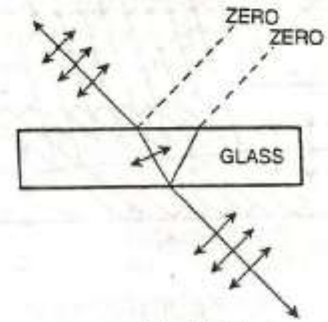


Fig. 10.10

The light component vibrating at right angles to the plane of incidence is reflected. After about 100 reflections at the Brewster window, the transmitted beam will have 50% of the intensity of the incident beam and it will be completely plane polarized. The net effect of this type of arrangement is that half the amount of light intensity has been discarded and the other half is completely retained. Brewster's windows are used in gas lasers.

10.8 POLARIZATION BY REFRACTION

It is found that at a single glass surface or any similar transparent medium, only a small fraction of the incident light is reflected.

For glass ($\mu = 1.5$) at the polarizing angle, 100% of the light vibrating parallel to the plane of incidence is transmitted whereas for the perpendicular vibrations only 85% is transmitted and 15% is reflected. Therefore, if we use a **pile of plates** and the beam of ordinary light is incident at the polarizing angle on the pile of plates, some of the vibrations perpendicular to the plane of incidence are reflected by the first plate and the rest are transmitted through it. When this beam of light is reflected by the second plate, again some of the vibrations perpendicular to the

Intensity of the ordinary ray

$$I_0 = A^2 \sin^2 \theta$$

$$\frac{I_E}{I_0} = \frac{A^2 \cos^2 \theta}{A^2 \sin^2 \theta} = \frac{\cos^2 \theta}{\sin^2 \theta}$$

Here $\theta = 30°$

$$\therefore \quad E = \frac{I_E}{I_0} = 3$$

10.10 DOUBLE REFRACTION

Erasmus Bartholinus discovered, in 1669, that when a ray of light is refracted by a crystal of calcite it gives two refracted rays. This phenomenon is called **double refraction**. Calcite or Iceland spar is crystallised calcium carbonate ($Ca\,CO_3$) and was found in large quantities in Iceland as very large transparent crystals. Due to this reason calcite is also known as Iceland spar. It crystallises in many forms and can be reduced by cleavage or breakage into a rhombohedron, bounded by six parallelograms with angles equal to 102° and 78° (more accurately 101° 55′ and 78° 5′).

Optic Axis. At two opposite corners *A* and *H*, of the rhombohedron all the angles of the faces are obtuse [Fig. 10.13 (*a*)]. These corners *A* and *H* are known as the blunt corners of the crystal. A line drawn through *A* making equal angles with each of the three edges gives the direction of the optic axis. In fact any line parallel to this line is also an optic axis. Therefore, optic axis is not a line but it is a direction. Moreover, it is not defined by joining the two blunt corners. Only in a special case, when the three edges of the crystal are equal, the line joining the two blunt corners *A* and *H* coincides with the crystallographic axis of the crystal and it gives the direction of the optic axis [Fig. 10.13 (*b*)]. If a ray of light is incident along the optic axis or in a direction parallel to the optic axis, then

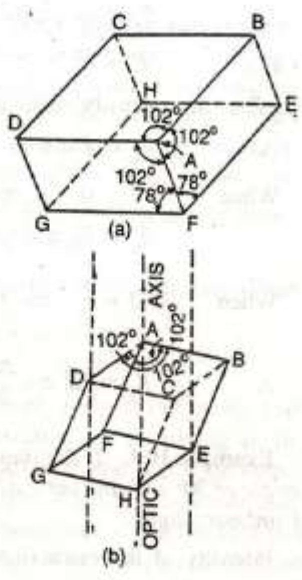


Fig. 10.13

it will not split into two rays. **Thus, the phenomenon of double refraction is absent when light is allowed to enter the crystal along the optic axis.**

The phenomenon of double refraction can be shown with the help of the following experiment :

Mark an ink dot on a piece of paper. Place a calcite crystal over this dot on the paper. Two images will be observed. Now rotate the crystal

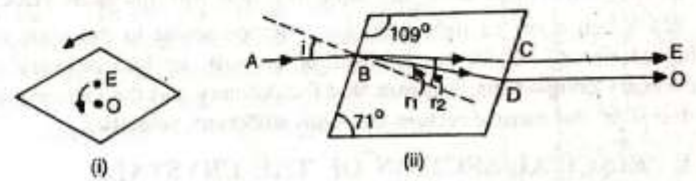


Fig. 10.14

slowly as shown in Fig. 10.14 (*i*). Place your eye vertically above the crystal. It is found that one image remains stationary and the second image rotates with the rotation of the crystal. The stationary image is known as the ordinary image while the second one is known as the extraordinary image.

When a ray of light *AB* is incident on the calcite crystal making an angle of incidence = *i*, it is refracted along two paths inside the crystal, (*i*) along *BC* making an angle of refraction = r_2 and (*ii*) along *BD* making an angle of refraction = r_1. These two rays emerge out along *DO* and *CE* which are parallel [Fig. 10.14 (*ii*)].

The ordinary ray has a refractive index $\mu_0 = \dfrac{\sin i}{\sin r_1}$ and the extraordinary ray has a refractive index $\mu_e = \dfrac{\sin i}{\sin r_2}$. It is found that the ordinary ray obeys the laws of refraction and its refractive index is constant. In the case of the extraordinary ray, its refractive index varies with the angle of incidence and it is not fixed.

In the case of calcite $\mu_0 > \mu_e$ because r_1 is less than r_2 [Fig. 10.14 (*ii*)]. Therefore the **velocity of light for the ordinary ray inside the crystal will be less** compared to the velocity of light for the extraordinary ray. In calcite, the extraordinary ray travels faster as compared to the ordinary ray. Moreover, the velocity of the extraordinary ray is different in different directions because its refractive index varies with the angle of incidence.

It has been found that both the rays are plane polarized. The vibrations of the ordinary ray are perpendicular to the principal section of the crystal while the vibrations of the extraordinary ray are in the plane of the principal section of the crystal. Thus, the two rays are **plane polarised**, their vibrations being at right angles to each other.

Special Cases. (1) It should be remembered that a ray of light is not split up into ordinary and extraordinary components when it is incident on calcite parallel to its optic axis. In this case, the ordinary and the extraordinary rays travel along the same direction with the same velocity.

(2) When a ray of light is incident perpendicular to the optic axis on the calorie crystal, the ray of light is not split up into ordinary and extraordinary components. It means that the ordinary and the extraordinary rays travel in the same direction but with **different velocities**.

10.11 PRINCIPAL SECTION OF THE CRYSTAL

A plane which contains the optic axis and is perpendicular to the opposite faces of the crystal is called the **principal section** of the crystal. As a crystal has six faces, therefore, for every point there are three principal sections. A principal section always cuts the surface of a calcite crystal in a parallelogram with angles 109° and 71°

10.12 PRINCIPAL PLANE

A plane in the crystal drawn through the optic axis and the ordinary ray is defined as the principal plane of the ordinary ray, Similarly, a plane in the crystal drawn through the optic axis and the extraordinary ray is defined as the principal plane of the extraordinary ray. In general, the two planes do not coincide. In a particular case, when the plane of incidence is a principal section then the principal section of the crystal and the principal planes of the ordinary and the extraordinary rays coincide.

10.13 NICOL PRISM

It is an optical device used for producing and analysing plane polarized light. It was invented by William Nicol, in 1828, who was an expert in cutting and polishing gems and crystals. We have discussed that when a beam of light is transmitted through a calcite crystal, it breaks up into two rays : (1) the ordinary ray which has its vibrations perpendicular to the principal section of the crystal and (2) the extraordinary ray which has its vibrations parallel to the principal section.

The **nicol prism** is made in such a way that it eliminates one of the two rays by total internal reflection. It is generally found that the ordinary ray is eliminated and only the extraordinary ray is transmitted through the prism.

A calcite crystal whose length is three times its breadth is taken. Let $A'BCDEFG'H$ represent such a crystal having A' and G' as its blunt corners and $A'CG'E$ is one of the principal sections with $\angle A'CG' = 70°$.

The faces $A'BCD$ and $EFG'H$ are ground in such a way that the angle ACG becomes = 68° instead of 71°. The crystal is then cut along the plane $AKGL$ as shown in Fig. 10.15. The two cut surfaces are grounded and polished optically flat and then cemented together by Canada balsam whose refractive index lies between the refractive indices for the ordinary and the extraordinary rays for calcite.

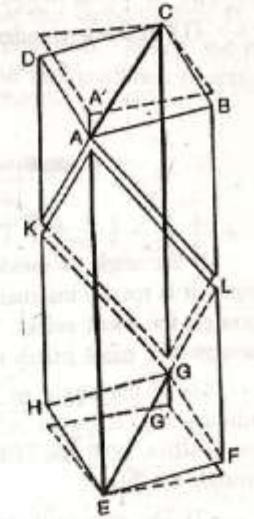


Fig. 10.15

Refractive index for the ordinary

$$\mu_0 = 1.658$$

Refractive index for Canada balsam

$$\mu_B = 1.55$$

Refractive index for the extraordinary $\mu_E = 1.486$

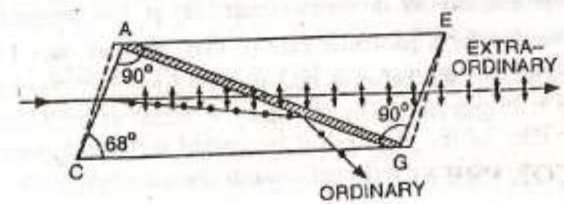


Fig. 10.16

In Fig. 10.16, the section $ACGE$ of the crystal is shown, The diagonal AG represents the Canada balsam layer in the plane $ALGK$ of Fig. 10.15.

It is clear that Canada balsam acts as a rarer medium for an ordinary ray and it acts as a denser medium for the extraordinary ray. Therefore, when the ordinary ray passes from a portion of the crystal into the layer of Canada balsam it passes from a denser to a rarer medium. When the angle of incidence is greater than the critical angle, the ray is totally internally reflected and is not transmitted. The extraordinary ray is not affected and is therefore transmitted through the prism. The working of the prism is clear from the following cases :-

(1) Refractive index for ordinary ray with respect to Canada balsam

$$= \mu = \frac{1.658}{1.550}$$

$$\therefore \quad \sin\theta = \frac{1}{\mu} = \frac{1.550}{1.658}$$

$$\therefore \quad \theta = 69^\circ$$

If the angle of incidence for the ordinary ray is more than the critical angle, it is totally internally reflected and only the extraordinary ray passes through the nicol prism. Therefore, a ray of unpolarized light on passing through the nicol prism in this position becomes plane-polarized.

(2) If the angle of incidence is less than the critical angle for the ordinary ray, it is not reflected and is transmitted through the prism. In this position both the ordinary and the extraordinary rays are transmitted through the prism.

(3) The extraordinary ray also has a limit beyond which it is totally internally reflected by the Canada balsam surface. The refractive index for the extraordinary ray = 1.486 when the extraordinary ray is travelling at right angles to the direction of the optic axis. If the extraordinary ray travels along the optic axis, its refractive index is the same as that of the ordinary ray and it is equal to 1.658. Therefore, depending upon the direction of propagation of the extraordinary ray μ_e lies between 1.486 and 1.658. Therefore for a particular case μ_e may be more than 1.55 and the angle of incidence will be more than the critical angle. Then, the extraordinary ray will also be totally internally reflected at the Canada balsam layer. The sides of the nicol prism are coated with black paint to absorb the ordinary rays that are reflected towards the sides by the Canada balsam layer.

10.14 NICOL PRISM AS AN ANALYSER

Nicol prism can be used for the production and detection of plane polarizer light.

When two nicol prisms P_1 and P_2 are placed adjacent to each other as shown in Fig. 10.17 (*i*), one of them acts as a polarizer and the other acts as an analyser. Fig. 10.17 (*i*) shows the position of two parallel nicols and only the extraordinary ray passes through both the prisms.

If the second prism P_2 is gradually rotated, the intensity of the extraordinary ray decreases in accordance with Malus Law and when

the two prisms are crossed [*i.e.*, when they are at right angles to each other, Fig. 10.16 (*ii*), then no light comes out of the second prism P_2. It means that light coming out of P_1 is plane polarized. When the polarized extraordinary ray enters the prism P_2 in this positions it acts as

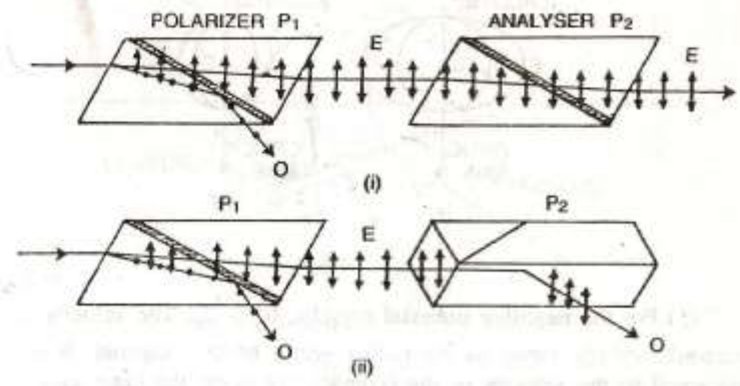


Fig. 10.17

an ordinary ray and is totally internally reflected by the Canada balsam layer and so no light comes out of P_2. Therefore, the prism P_1 produces plane-polarized light and the prism P_2 detects it.

Hence P_1 and P_2 are called the polarizer and the analyser respectively. The combination of P_1 and P_2 is called a polariscope.

10.15 HUYGENS EXPLANATION OF DOUBLE REFRACTION IN UNIAXIAL CRYSTALS

Huygens explained the phenomenon of double refraction with the help of his principle of secondary wavelets. A point source of light in a double refracting medium is the origin of two wavefronts. For the ordinary ray, for which the velocity of light is the same in all directions the wavefront is spherical. For the extraordinary ray, the velocity varies with the direction and the wavefront is an ellipsoid of revolution. **The velocities of the ordinary and the extraordinary rays are the same along the optic axis.**

Consider a point source of light S in a calcite crystal [Fig. 10.18.(*a*)]. The sphere is the wave surface for the ordinary ray and the ellipsoid is the wave surface for the extraordinary ray. The ordinary wave surface lies within the extraordinary wave surface. Such crystals are known as **negative crystals**. For crystals like quartz, which are known as positive crystals, the extraordinary wave surface lies within the ordinary wave surface [Fig. 10.18 (*b*)].

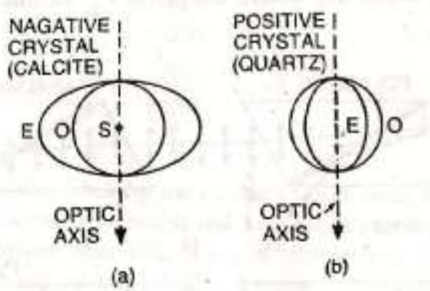


Fig. 10.18

(1) For the **negative uniaxial** crystals, $\mu_0 > \mu_E$. The velocity of the extraordinary ray varies as the radius vector of the ellipsoid. It is least and equal to the velocity of the ordinary ray along the optic axis but it is maximum at right angles to the direction of the optic axis.

(2) For the **positive uniaxial** crystals $\mu_E > \mu_0$. The velocity of the extraordinary ray is least in a direction at right angles to the optic axis. It is maximum and is equal to the velocity of the ordinary ray along the optic axis. Hence, from Huygens' theory, the wavefronts or surfaces in uniaxial crystals are a sphere and an ellipsoid and there are two points where these two wavefronts touch each other. The direction of the line joining these two points (Where the sphere and the ellipsoid touch each other) is the **optic axis.**

10.16 OPTIC AXIS IN THE PLANE OF INCIDENCE AND INCLINED TO THE CRYSTAL SURFACE

(*a*) **Oblique incidence.** *AB* is the incident plane wavefront of the rays falling obliquely on the surface *MN* of the negative crystal. The crystal is cut so that the optic axis is in the plane of incidence and is in the direction shown in Fig. 10.19. O_1 is the spherical secondary wavefront for the ordinary ray and E_1 is the ellipsoidal secondary wavefront for the extraordinary ray. *CP* is the tangent meeting the spherical wavefront at *P* and *CQ* is the tangent meeting the ellipsoidal wavefront at *Q*.

According to Huygens' construction, by the time the incident wave reaches from *B* to *C*, the ordinary ray travels the distance *AP* and the extraordinary ray travels the distance *AQ*. Suppose, the velocity of light

in air is V_a and the velocities of light for the ordinary ray along *AP* and the extraordinary ray along *AQ* are V_0 and V_e respectively. In this case,

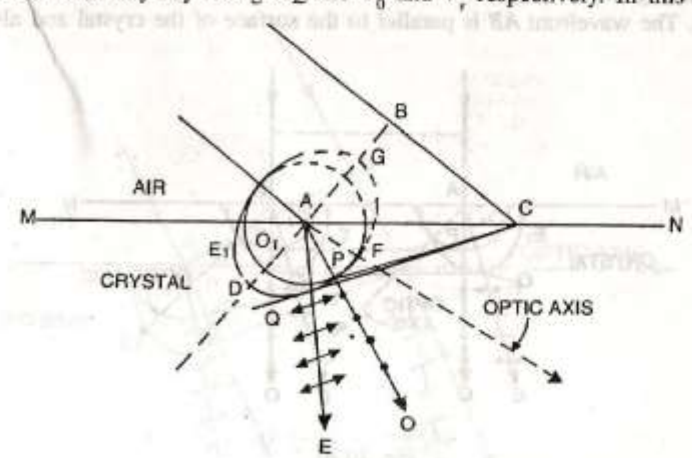


Fig. 10.19

$$\frac{BC}{V_a} = \frac{AP}{V_0} = \frac{AQ}{V_e} \quad ...(i)$$

Therefore $$AP = \frac{BC.V_0}{V_a} = \frac{BC}{\mu_0} \quad ...(ii)$$

and $$AQ = \frac{BC.V_e}{V_a} = \frac{BC}{\mu_e} \quad ...(iii)$$

Here, μ_0 and μ_e are the refractive indices for the ordinary and the extraordinary rays along *AP* and *AQ* respectively. In Fig. 10.19, *CP* and *CQ* are the ordinary and the extraordinary refracted plane wavefronts respectively in the crystal. Therefore, the ordinary and the extraordinary rays travel with different velocities along different direction. Here, the semi-major axis of the ellipsoid is $\frac{BC}{\mu_E}$ and the semi-minor axis is $\frac{BC}{\mu_0}$, where μ_E is the principal refractive index for the extraordinary ray and

$$\mu_E < \mu_e < \mu_0$$

Note. The direction *AE* of the extraordinary ray is not perpendicular to the tangent *CQ* , whereas the direction *AO* of the ordinary ray is perpendicular to the tangent *CP*.

index is maximum. **Therefore, the principal refractive index for the positive uniaxial crystal is the ratio of the velocity of light in vacuum to the minimum velocity of the extraordinary ray.**

For a negative crystal of calcite, $\mu_0 = 1.658$ and $\mu_E = 1.486$. Therefore, the ratio of the major to the minor axis of the wave surface of the extraordinary ray is 1.658 : 1.486.

For a positive crystal of quartz, $\mu_0 = 1.544$ and $\mu_E = 1.553$. Therefore, the ratio of the major to the minor axis of the wave surface of the extraordinary ray is, 1.553 : 1.544.

10.21 EXPERIMENTAL DETERMINATION OF REFRACTIVE INDEX

For determining the refractive index for the extraordinary ray a calcite crystal is cut in the form of a prism with the optic axis perpendicular to the refracting edge of the prism and perpendicular to the base BC [Fig. 10.27 (*a*)]. It can also be cut with the optic axis parallel to the

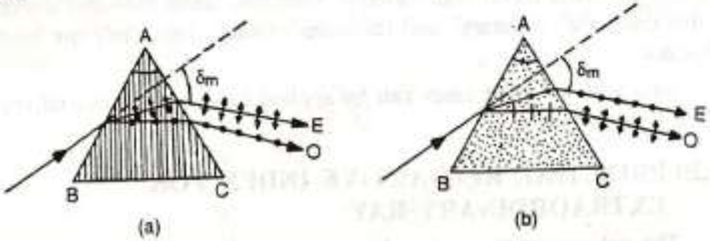


Fig. 10.27 (*a*) *Optic axis per perpendicular to the refracting edge.*
(*b*) *Optic axis parallel to the refracting edge.*

refracting edge of the prism [Fig. 10.27 (*b*)]. The prism is placed on the spectrometer table and is adjusted for the minimum deviation position for the extraordinary rays. The angle of minimum deviation δ_m is determined and the principal refractive index for the extraordinary ray is calculated from the relation.

$$\mu_E = \frac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\dfrac{A}{2}}$$

For a given wavelength, the ordinary and the extraordinary rays are separated while passing through the prism. Therefore, the angle of minimum deviation for the ordinary ray can be measured and thus its refractive index can be calculated.

10.22 DOUBLE IMAGE POLARIZING PRISMS

Nicol prism cannot be used with ultraviolet light on account of the Canada babam layer which absorbs these rays. Sometimes, it is also desirable to have both the ordinary and the extraordinary rays widely separated. For this purpose two prisms *viz.* (*i*) Rochon prism and (*ii*) Wollaston prism are used.

(1) **Rochon Prism.** It consists of two prisms ABC and BCD (of quartz or calcite) cut with their optic axes as shown in Fig. 10.28. The prism ABC is cut such that the optic axis is parallel to the face AB and the incident light. The prism BCD has the opticaxis perpendicular to the plane of incidence. Light incident normally on the face AC of the prism passes undeviated up to the boundary BC. In the prism BCD, the ordinary ray passes undeviated. If the prisms are made of quartz, the extraordinary ray is deviated as shown in Fig. 10.28. In the case of calcite, the extraordinary will be deviated to the other side. The prisms ABC and BCD are cemented together by glycerine or castor oil. Here, the ordinary emergent beam is achromatic whereas the extraordinary beam is chromatic.

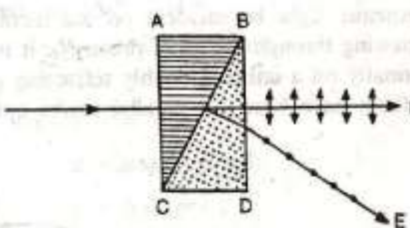


Fig. 10.28

(2) **Wollaston prism.** It consists of two prisms ABC and BCD of quartz or calcite cut with their optic axes as shown in Fig. 10.29. They are cemented together by glycerine or caster oil.

A ray of light is incident normally on the face AC of the prism ABC. The ordinary and the extraordinary rays travel along the same direction but with different speeds. After passing BC the ordinary ray behaves as the extra ordinary and the extra ordinary behaves as the ordinary while passing through the prism BCD. One ray is bent towards the normal while the other is bent away from the normal. In quartz $\mu_E > \mu_0$. Therefore, the ordinary ray while passing the boundary BC is refracted towards the normal as an extraordinary ray while the extraordinary ray is refracted away from the normal as an ordinary ray as shown in Fig. 10.29. If the prisms are made from calcite, the directions of the ordinary and the extraordinary rays are interchanged. While coming out of the face BD of the prism, the ordinary and the extraordinary rays are diverged. The prism

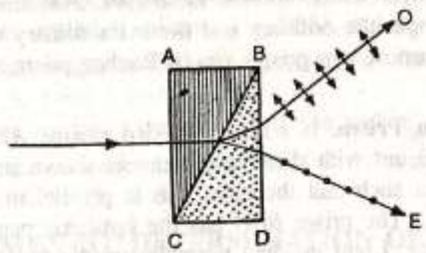


Fig. 10.29

is useful in determining the percentage of polarization in a partially polarized beam. Double image prisms are used in spectrophotometers and pyrometers.

10.23 ELLIPTICALLY AND CIRCULARLY POLARISED LIGHT

Let monochromatic light be incident on the nicol prism N_1 [Fig. 10.30 (*a*)]. After passing through the nicol prism N_2, it is plane-polarized and is incident normally on a uniaxial doubly refracting crystal P (calcite or quartz) whose faces have been cut parallel to the optic axis. The vibrations

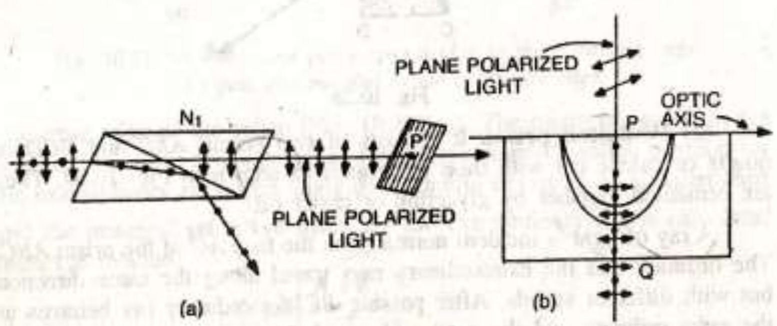


Fig. 10.30

of the plane-polarized light incident on the crystal are shown in Fig. 10.30 (*b*). The plane polarized light on entering the crystal is split up into two components, ordinary and extraordinary. Both the rays, in this case, travel along the same direction but, with different velocities. When the rays have

travelled through the thickness d in the crystal, a phase difference δ is introduced between them.

Theory. Suppose the amplitude of the incident plane polarized light on the crystal is A and it makes an angle θ with the optic axis (Fig. 10.31). Therefore, the amplitude of the ordinary ray vibrating along PO is $A \sin\theta$ and the amplitude of the extraordinary ray vibrating along PE is $A\cos\theta$. Since a phase difference δ is introduced between the two rays, after passing thorough a thickness d of the crystal, the rays after comming out of the crystal can be represented in terms of two simple harmonic motions, at right angles to each other and having a phase difference.

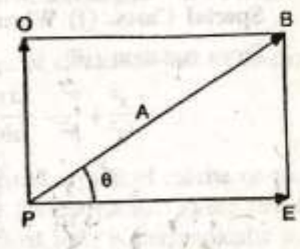


Fig. 10.31

$\therefore$ For the extraordinary ray,

$$x = A\cos\theta . \sin(\omega t + \delta)$$

For the ordinary ray,

$$y = A\sin\theta . \sin\omega t$$

Take, $A\cos\theta = a$

and $A\sin\theta = b$

$$x = a\sin(\omega t + \delta) \quad ...(i)$$

$$y = b\sin\omega t \quad ...(ii)$$

From (*ii*)

$$\frac{y}{b} = \sin\omega t$$

and

$$\cot\omega t = \sqrt{1 - \frac{y^2}{b^2}}$$

$\sqrt{1-\sin^2\theta}$

$$\frac{x}{a} = \sin\omega t\cos\delta + \cos\omega t\sin\delta$$

$\sin(A+B) = \sin A\cos B + \cos$

$$\frac{x}{a} = \frac{y}{b}\cos\delta + \sqrt{1 - \frac{y^2}{b^2}} . \sin\delta$$

$$\frac{x}{a} - \frac{y}{b}\cos\delta = \sqrt{1 - \frac{y^2}{b^2}} . \sin\delta$$

Squaring and rearranging

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy}{ab}\cos\delta = \sin^2\delta \quad ...(iii)$$

This is the general equation of an ellipse.

Special Cases. (1) When $\delta = 0$ $\sin\delta = 0$ and $\cos\delta = 1$

From equation *(iii)*

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy}{ab} = 0$$

$$\left(\frac{x}{a}-\frac{y}{b}\right)^2 = 0$$

or $$y = \frac{bx}{a}$$

This is the equation of a straight line. Therefore, the emergent light will be plane polarized (Fig. 10.32).

(2) When $\delta = \frac{\pi}{2}$, $\cos\delta = 0, \sin\delta = 1$

From equation *(iii)*

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$$

This represents the equation of a symmetrical ellipse. The emergent light in this case will be **elliptically polarized** provided $a \neq b$.

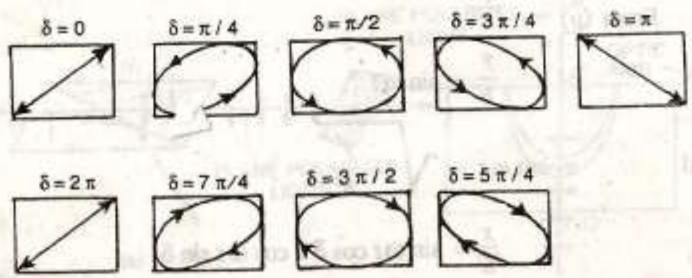


Fig. 10.32

(3) When $\delta = \frac{\pi}{2}$ and $a = b$

From equation *(iii)*,

$$x^2+y^2 = a^2$$

This represents the equation of circle of radius a. The emergent light will be **circularly polarized**. Here the vibrations of the incident-plane-polarized light on the crystal make an angle of 45° with the direction of the optic axis.

(4) For $\delta = \pi/4$ or $7\pi/4$, the shape of the ellipse will be as shown in Fig. 10.32.

(5) For all other values of δ, the nature of vibrations will be as shown in Fig. 10.32.

10.24 QUARTER WAVE PLATE

It is a plate of doubly refracting uniaxial crystal of calcite or quartz of suitable thickness whose refracting faces are cut parallel to the direction of the optic axis. The incident plane-polarized light is perpendicular to its surface and the ordinary and the extraordinary rays travel along the same direction with different velocities. If the thickness of the plate is t and the refractive indices for the ordinary and the extraordinary rays are μ_0 and μ_E respectively, then the path difference introduced between the two rays is given by :

For negative crystals, path difference $= (\mu_0-\mu_E)\,t$

For positive crystals, path difference $= (\mu_E-\mu_0)\,t$

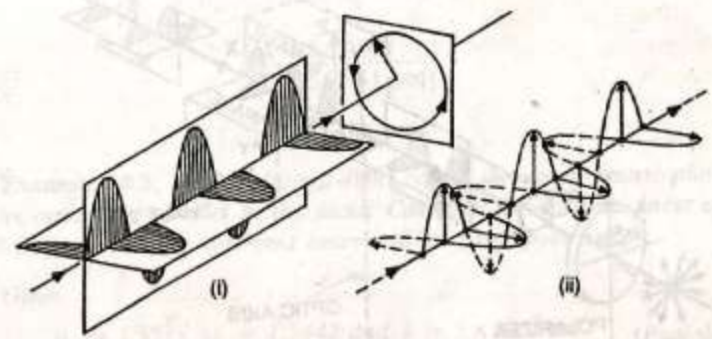


Fig. 10.33

To produce a path difference of $\frac{\lambda}{4}$, in calcite

$$(\mu_0-\mu_E)\,t = \frac{\lambda}{4}$$

or $$t = \frac{\lambda}{4(\mu_0-\mu_E)} \quad ...(i)$$

and in the case of quartz,

$$t = \frac{\lambda}{4(\mu_E - \mu_0)} \quad ...(ii)$$

If the plane-polarized light, whose plane of vibration is inclined at an angle of 45° to the optic axis, is incident on a quarter wave plate, the emergent light is circularly polarized (Fig. 10.33).

10.25 HALF WAVE PLATE

This plate is also made from a doubly refracting uniaxial crystal of quartz or calcite with its refracting faces cut parallel to the optic axis. The thickness of the plate is such that the ordinary and the extraordinary rays have a path difference = $\frac{\lambda}{2}$ after passing through the crystal.

For negative crystals, path difference = $(\mu_0 - \mu_E)\, t$

For positive crystals, path difference = $(\mu_E - \mu_0)\, t$

To produce a path difference of $\frac{\lambda}{2}$ in calcite,

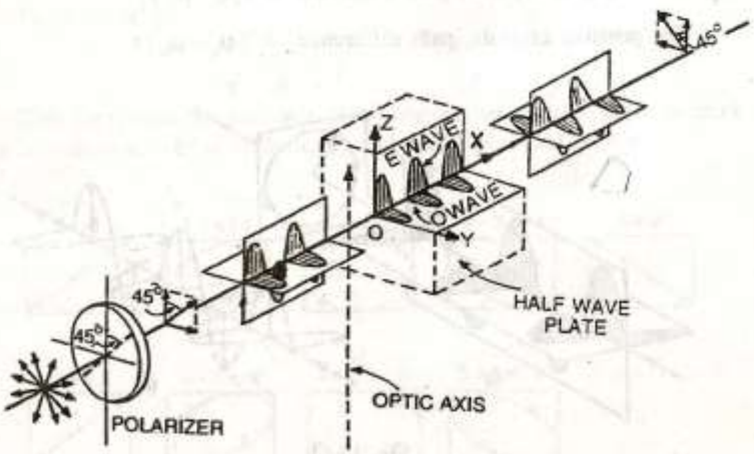


Fig. 10.34

$$(\mu_0 - \mu_E)\, t = \frac{\lambda}{2}$$

or

$$t = \frac{\lambda}{2(\mu_0 - \mu_E)} \quad ...(i)$$

and in the case of quartz,

$$t = \frac{\lambda}{2(\mu_E - \mu_0)} \quad ...(ii)$$

When plane-polarized light is incident on a half-wave plate such that it makes an angle of 45° with the optic axis, a path difference of $\frac{\lambda}{2}$ is introduced between the extraordinary and the ordinary rays. The emergent light is plane-polarized and the direction of polarization of the linear incident light is rotated through 90° as shown in Fig. 10.31. Thus, **a half wave plate rotates the azimuth of a beam of plane polarized light** by 90°, provided the incident light makes an angle of 45° with the optic axis of the half wave plate.

Example 10.2. *Calculate the thickness of a half wave plate of quartz for a wavelength of 5000 Å. Here μ_E = 1.553 and μ_0 = 1.544.* *(Delhi)*

For a half wave plate,

$$t = \frac{\lambda}{2[\mu_E - \mu_0]}$$

Here $\lambda = 5000\text{ Å} = 5 \times 10^{-5}$ cm

$\mu_E = 1.553,\ \mu_0 = 1.544,\ t = ?$

$$\therefore \quad t = \frac{5 \times 10^{-5}}{2[1.553 - 1.544]}$$

or $\quad t = \mathbf{2.78 \times 10^{-3}}$ **cm**

Example 10.3. *Plane-polarized light passes through a quartz plate with its optic axis parallel to the faces. Calculate the least thickness of the plate for which the emergent beam will be plane-polarized.*

Given

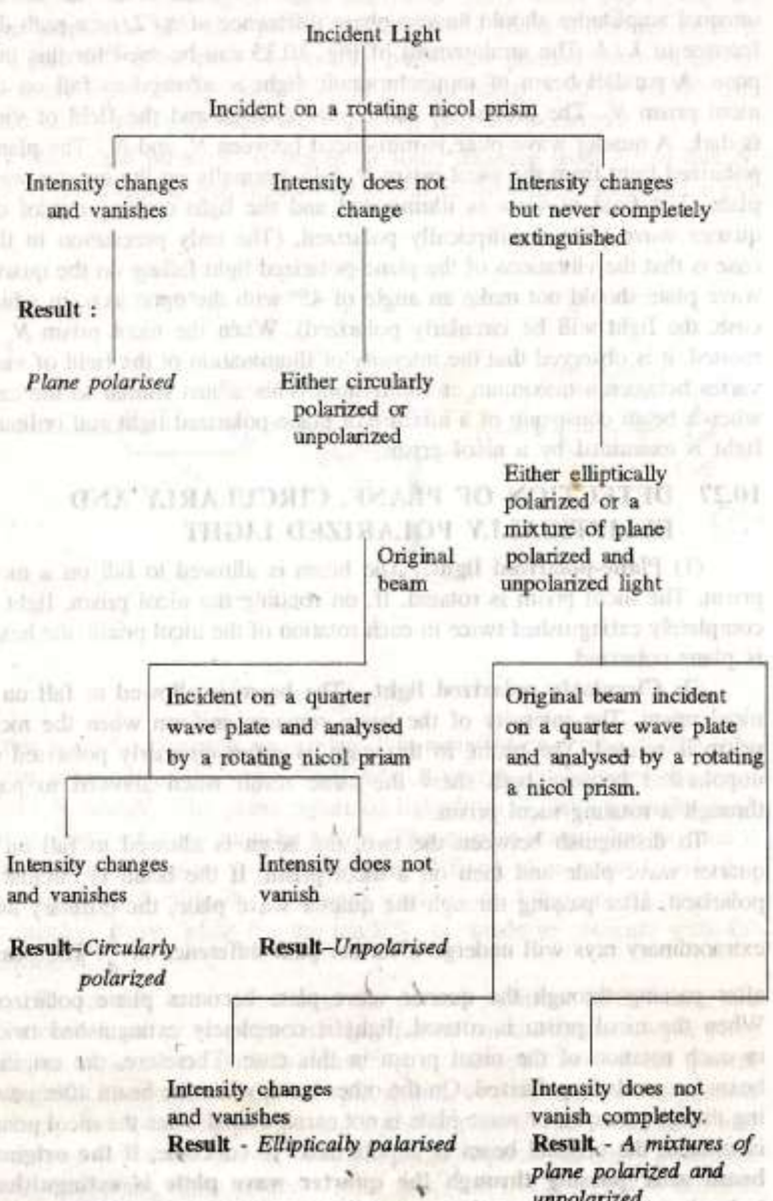
$\mu_E = 1.5533,\ \mu_0 = 1.5442$ and $\lambda = 5 \times 10^{-5}$ cm *(Punjab)*

$$t = \frac{\lambda}{2(\mu_E - \mu_0)}$$

$$= \frac{5 \times 10^{-5}}{2(1.5533 - 1.5442)}$$

$$= \mathbf{2.75 \times 10^{-3}\ cm}$$

Table 10.1



(3) **Elliptically polarized light.** The beam is allowed to fall on a nicol prism. If the beam is elliptically polarised, the intensity varies from a maximum to a minimum value when the nicol prism is rotated. The maximum or minimum intensity depends on the condition whether the principal plane of the nicol prism is parallel to the major axis or to the minor axis of the ellipse. This is just similar to the case when a beam consisting of a mixture of unpolarized light and polarized light is allowed to pass through a rotating nicol prism.

To distinguish between the two, the original beam is allowed to fall on a quarter wave plate and then on a nicol prism. If the beam is elliptically polarized, the ordinary and the extraordinary rays will undergo a further path difference of $\frac{\lambda}{4}$. The beam after passing through the quarter wave plate becomes plane polarized and is completely extinguished twice in each rotation of the nicol prism. Therefore, the original beam is elliptically polarized. On the other hand, when the original beam after passing through the nicol prism is not completely extinguished when studied by a rotating nicol prism, the original beam is a mixture of plane polarized and unpolarized light.

Example 10.12. *Elliptically polarized light falls on a quarter wave plate normally. Explain the nature of the emergent light if the major axis makes the following angles with the principal plane of the quarter wave plate : (i) zero degree (ii) 30°. [Bombay]*

(1) When elliptically polarized light passes through a quarter wave plate and the major axis makes an angle equal to zero degree with the principal plane, the beam will become plane polarized.

(2) In this case, the beam remains elliptically polarised with different intensity.

10.28 BABINET'S COMPENSATOR

A quarter wave plate or a half wave plate produces only a fixed path difference between the ordinary and the extraordinary rays and can be used only for light of a particular wavelength. For different wavelengths, different quarter wave plates are to be used. To avoid this difficulty, Babinet designed a compensator by means of which a desired path difference can be introduced.

It consists of two wedge-shaped sections A and B of quartz (Fig. 10.36). The optic axis is lengthwise in A and transverse in B. The outer faces of the compensator are parallel to the optic axis. Therefore, the ordinary and the extraordinary rays travel with different velocities along the same direction inside the compensator. Moreover, the extraordinary ray in A behaves as ordinary in B while the ordinary in A

behaves as extraordinary in *B*. Suppose a plane polarized parallel beam of light is incident normally at the point *C* of the Babinet's compensator. The beam is split up into extraordinary and ordinary rays. The path difference introduced between them after they have travelled a distance *CD* in *A* is $(\mu_E - \mu_0)\, t_1$. In *B*, the extraordinary ray behaves as ordinary and vice versa. Therefore, the path difference introduced by *B* is $(\mu_0 - \mu_E)\, t_2$.

Therefore, the resultant path difference

$$= (\mu_E - \mu_0)\,(t_1 - t_2)$$

The crystals *A* and *B* are mounted such that *A* is fixed and *B* can slide along the surface of *A* with the help of a rack and pinion arrangement. In this way $(t_1 - t_2)$ can be made to have any desired value. Hence any path difference can be introduced with the help of the Babinet's compensator and it can be used for light of any wavelength.

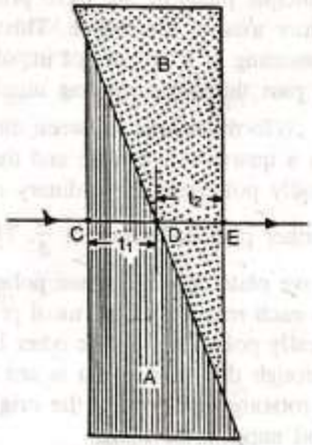


Fig. 10.36

10.29 DICHROISM

There are certain crystals and minerals which are doubly refracting and have the property of absorbing the ordinary and the extraordinary rays unequally. In this way, plane polarized light is produced. The crystals showing this property are said to be **dichroic** and the phenomenon is known as dichroism. Tourmaline is a dichroic crystal and absorbs the ordinary ray completely as shown in Fig. 10.37. The ordinary ray is completely absorbed while the extraordinary ray is partly absorbed and so it emerges.

Herapath (English), in 1852, discovered a synthetic crystalline material iodo-sulphate of quinine known as herapathite which is a dichroic crystal. It transmits linerarly polarized light of all colours and wavelengths. But these crystals are not stable and are affected by slight strain. In 1934, Land developed a polarizer known as a polaroid in the form of large sheets.

Herapathite crystals are embedded in a volatile viscous medium and the crystals are aligned with their optic axes parallel. They are prevented

from shattering. There are a number of methods of preparing polaroid sheets. In one process, the dichroic crystals are embedded and arranged with their optic axes parallel in cellulose acetate. A more recent type is prepared by taking a sheet of polyvinyl alcohol and subjecting it to a large strain. In this way, the molecules are oriented parallel to the strain and

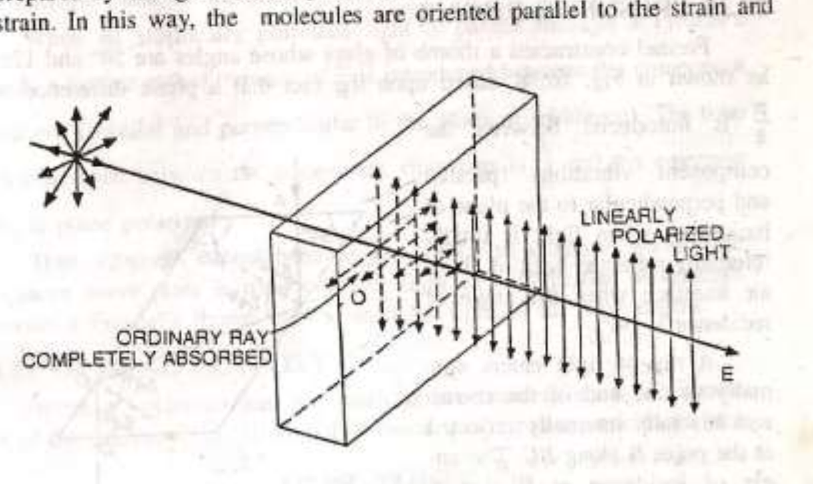


Fig. 10.37

the material becomes doubly refracting. It acts as a dichroic crystal when strained with iodine. The polaroid sheets are placed between glass plates so that they are not spoiled. When the two pieces of polaroids are

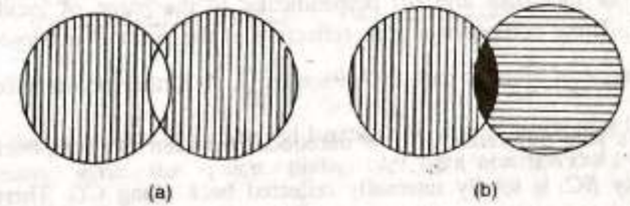


Fig. 10.38. Polaroids

uncrossed, the emergent beam is plane polarized, Fig. 10.38 (a). When the two polaroids are crossed, there is perfect extinction of light, Fig. 10.38 (b).

Uses of Polaroids. Polaroids are widely used as polarizing sun glasses. Polaroid films are used to produce three-dimensional moving pictures. They are used to eliminate the head light glare in motor cars. They are also used to improve the colour contrasts in old oil paintings and as glass windows in trains and aeroplanes. In aeroplanes, one of the polaroids is fixed while the other can be rotated to control the amount of light coming inside.

10.30 FRESNEL'S RHOMB

Fresnel constructed a rhomb of glass whose angles are 54° and 126° as shown in Fig. 10.38, based upon the fact that a phase difference of $\frac{\pi}{4}$ is introduced between the component vibrations (parallel and perpendicular to the plane of incidence) when light is totally internally reflected back at glass-air interface when the angle of incidence is 54°.

A ray of light enters normally at one end of the rhomb and is totally internally reflected at the point B along BC. The angle of incidence at B is 54°, which is more than the critical angle of glass. Let the incident light be plane polarized and let the vibrations make an angle of 45° with the plane of incidence. Its components (*i*) parallel to the plane of incidence and (*ii*) perpendicular to the plane of incidence are equal. These components after reflection at the point B undergo a phase difference of $\frac{\pi}{4}$ or a path difference of $\frac{\lambda}{8}$. A further phase difference of $\frac{\pi}{4}$ or a path difference of $\frac{\lambda}{8}$ is introduced between the components when the ray BC. is totally internally reflected back along CD. Therefore the final emergent ray DE has two components, vibrating at right angles to each other and they have a path difference of $\frac{\lambda}{4}$. Therefore, the emergent light DE is circularly polarized. Fresnel's rhomb works similar to a quarter wave plate.

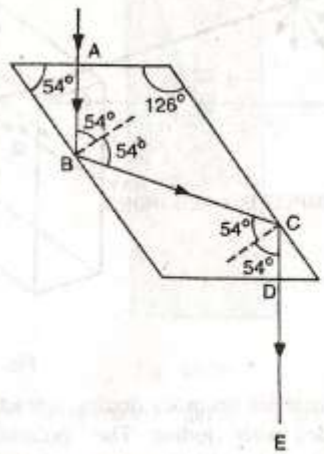


Fig. 10.39

If the light entering the Fresnel's rhomb is circularly polarized, a further path difference of $\frac{\lambda}{4}$ is introduced between the component vibrations.

The total path difference between the component vibrations is $\frac{\lambda}{2}$. Therefore the emergent light is plane polarised and its vibrations make an angle of 45° with the plane of incidence.

When an elliptically polarized light is passed through a Fresnel's rhomb, a further path difference of $\frac{\lambda}{4}$ is introduced between the component vibrations (parallel and perpendicular to the plane of incidence). The total path difference between the component vibrations is $\frac{\lambda}{2}$ and the emergent light is plane polarized.

Thus, Fresnel's rhomb behaves just similar to a quarter wave plate. A quarter wave plate is used only for light of a particular wavelength, whereas a Fresnel's rhomb can be used for light of all wavelengths.

10.31 OPTICAL ACTIVITY

When a polarizer and an analyser are crossed, no light emerges out of the analyser [Fig. 10.40 (*i*)]. When a quartz plate cut with its faces

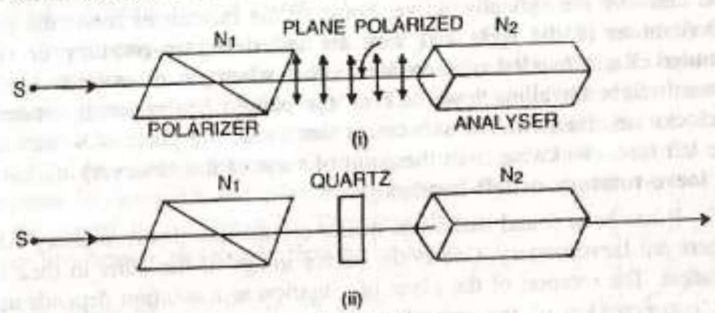


Fig. 10.40

parallel to the optic axis is introduced between N_1 and N_2 such that light falls normally upon the quartz plate, the light emerges out of N_2 [Fig. 10.40 (*ii*)].

The quartz plate turns the plane of vibration. The plane polarized light enters the quartz plate and its plane of vibration is gradually rotated as shown in Fig. 10.41.

The amount of rotation through which the plane of vibration is turned depends upon the thickness of the quartz plate and the wavelength of light. The action of turning the plane of vibration occurs inside the body of the plate and not on its surface. This phenomenon or the property of rotating the plane of vibration by certain crystals or substances is known as **optical**

activity and the substance is known as an optically active substance. It has been found that calcite does not produce any change in the plane of vibration of the plane polarised light. Therefore, it is not optically active.

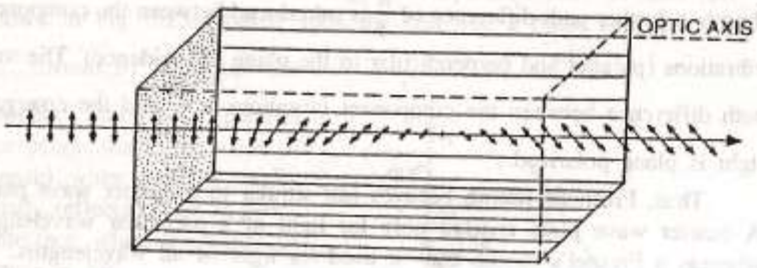


Fig. 10.41

Substances like sugar crystals, sugar solution, turpentine, sodium chlorate and cinnabar are optically active. Some of the substances rotate the plane of vibration to the right and they are called **dextro-rotatory** or **right handed.** Right handed rotation means that when the observer is looking towards light travelling towards him, the plane of vibration is rotated in a clockwise direction. The substances that rotate the plane of vibration to the left (anti-clockwise from the point of view of the observer) are known as **laevo-rotatory** or **left-handed**.

It has been found that some quartz crystals are dextro-rotatory while others are laevorotatory. One is the mirror image of the other in their orientation. The rotation of the plane of vibration in a solution depends upon the concentration of the optically active substance in the solution. This helps in finding the amount of cane sugar present in a sample of sugar solution.

10.32 FRESNEL'S EXPLANATION OF ROTATION

A linearly polarized light can be considered as a resultant of two circularly polarized vibrations rotating in opposite directions, with the same angular velocity. Fresnel assumed that a plane polarized light on entering a crystal along the optic axis is resolved into two circularly polarized vibrations rotating in opposite directions with the same angular velocity or frequency.

In a crystal like calcite, the two circularly polarized vibrations travel with the same angular velocity.

In Fig. 10.42, OL is the circularly polarised vector rotating in the anti-clockwise direction and OR is the circularly polarized vector rotating in the clockwise direction. The resultant vector of OR and OL is OA. According to Fresnel, when lineraly polarised light enters a crystal of calcite along the optic axis, the circularly polarized vibrations, rotating in opposite directions, have the same velocity. The resultant vibration will be along AB. Thus, crystals like calcite do not rotate the plane of vibration.

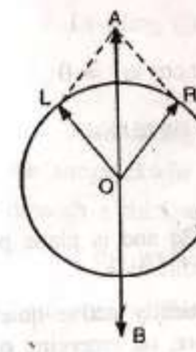


Fig. 10.42

In the case of quartz, the linearly polarized light, on entering the crystal is resolved into two circularly polarized vibrations rotating in opposite directions. In the case of a right-handed optically active crystal, the clockwise rotation travels faster while in a left-handed optically active crystal the anti-clockwise rotation travels faster.

Considering a right-handed quartz crystal (Fig. 10.43) the clockwise component travels a greater angle δ than the anticlockwise component when they emerge out of the crystal. The resultant of these two vectors OR and OL is along OA'. Therefore, the resultant vibrations are along $A'B'$. Before entering the crystal, the plane of vibration is along AB and after emerging out of the crystal it is along $A'B'$. Therefore, the plane of vibration has rotated through an angle $\frac{\delta}{2}$. The angle, through which the plane of vibration is rotated, depends upon the thickness of the crystal.

Analytical Treatment for Calcite. Circularly polarised light is the resultant of two rectangular components having a phase difference of $\frac{\pi}{2}$.

For clockwise circular vibrations,

$$x_1 = a \cos \omega t$$

$$x_2 = a \sin \omega t$$

Fig. 10.43

For anti-clockwise circular vibrations,

$$x_2 = -a\cos\omega t$$

$$y_2 = a\sin\omega t$$

Therefore, the resultant vibrations,

along the x-axis, $X = x_1 + x_2 = a\cos\omega t - a\cos\omega t = 0$

and along the y-axis, $Y = y_1 + y_2 = a\sin\omega t + a\sin\omega t$

$$= 2a\sin\omega t$$

Thus, the resultant vibration has an amplitude $2a$ and is plane polarized. The plane of vibration is along the original direction.

For Quartz. In the case of right-handed optically active quartz crystal, the clockwise vibration travels faster. Therefore, on emerging out of the crystal, the clockwise vibrations start from R and the anticlockwise vibrations start from L. The phase difference between them = δ.

For clockwise vibrations,

$$x_1 = a\cos(\omega t + \delta)$$

$$x_1 = a\sin(\omega t + \delta)$$

For anti-clockwise vibrations,

$$x_2 = -a\cos\omega t$$

$$y_2 = a\sin\omega t$$

Therefore the resultant displacements along the two axes are,

$$X = x_1 + x_2$$

$$= a\cos(\omega t + \delta) - a\cos\omega t$$

$$= 2a\sin\frac{\delta}{2}\sin\left(\omega t + \frac{\delta}{2}\right) \quad ...(i)$$

$$Y = y_1 + y_2$$

$$= a\sin(\omega t + \delta) + a\sin\omega t$$

$$= 2a\cos\frac{\delta}{2}\sin\left(\omega t + \frac{\delta}{2}\right) \quad ...(ii)$$

The resultant vibrations along the X-axis and Y-axis have the same phase. Therefore, the resultant vibration is plane polarized and it makes an angle $\frac{\delta}{2}$ with the original direction. Therefore, the plane of vibration has rotated through an angle $\frac{\delta}{2}$ on passing through the crystal.

Dividing (i) by (ii),

$$\tan\frac{\delta}{2} = \frac{X}{Y}$$

Also taking the refractive index of clockwise vibrations = μ_R and the anticlockwise vibration = μ_L, the optical path difference in passing through a thickness d of the crystal = $(\mu_L - \mu_R)\,d$.

If the wavelengths of light = λ,

then, phase difference $\quad \delta = \frac{2\pi}{\lambda} \times (\text{path difference})$

$$\delta = \frac{2\pi}{\lambda}(\mu_L - \mu_R)\,d$$

The plane of vibration is rotated through an angle

$$\frac{\delta}{2} = \frac{\pi}{\lambda}\left(\mu_L - \mu_R\right)d$$

In the case of left-handed optically active crystals,

$$\frac{\delta}{2} = \frac{\pi}{\lambda}(\mu_R - \mu_L)\,d$$

Example 10.3. *The rotation in the plane of polarization (λ = 5893 Å) in a certain substance is 10°/cm. Calculate the difference between the refractive indices for right and left circularly polarized light in the substance.* (Mysore)

Here $\quad \delta = \frac{2\pi}{\lambda}\left(\mu_R - \mu_L\right)d$

$$\mu_R - \mu_L = \left(\frac{\delta}{d}\right)\left(\frac{\lambda}{2\pi}\right)$$

Here $\quad \frac{\delta}{d} = 10°/\text{cm} = \frac{10 \times 2\pi}{360}$

$$= \frac{\pi}{36}\ \text{radian/cm}$$

$$\lambda = 5893 \times 10^{-8}\ \text{cm}$$

$\therefore \quad \mu_R - \mu_L = \frac{\pi}{36} \times \frac{5893 \times 10^{-8}}{2\pi} = \mathbf{8.185 \times 10^{-7}}$

10.33 FRESNEL'S EXPERIMENT

Fresnel showed that linearly polarized light on entering an optically active crystal is resolved into two circularly polarized vibrations. He arranged a number of negative and positive optically active quartz prisms as shown in Fig. 10.44.

Two circularly polarized beams were observed, one rotating to the right (clockwise) and the other rotating to the left (anticlockwise). The optic axis is parallel to the base of each prism. When plane polarized light in incident normally on the first crystal surface (*R*), the two component circular vibrations (clockwise and anticlockwise, travel along the same direction with different speeds. When the beam is incident on the sblique

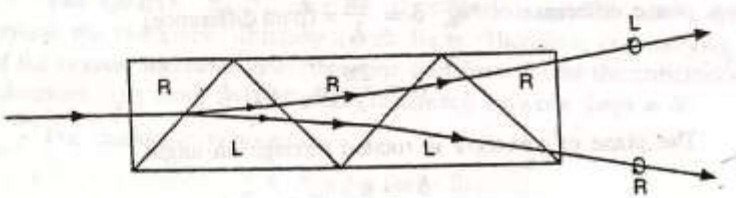


Fig. 10.44

surface of the second prism (*L*), the beam which was faster in the first prism becomes slower in the second prism and *vice versa*. Therefore, one beam is bent away from the normal and the other is bent towards the normal. The two beams are separated apart, while they travel through the prism *L*. Again at the boundary of the next prism (*R*), the speeds are interchanged and the beam that is bent towards the normal in prism *L*, is now bent away from the normal. Thus the two beams are separated more and more while passing through the arrangement. When the two beams emerge out, they are widely apart. When these beams are treated with a quarter wave plate and a nicol prism, both are found to be circularly polarized.

10.34 SPECIFIC ROTATION

Liquids containing an optically active substance *e.g.*, sugar solution, camphor in alcohol etc. rotate the plane of the linearly polarized light. The angle through which the plane polarized light is rotated depends upon (1) the thickness of the medium (2) concentration of the souution or density of the active substance in the solvent (3) wavelength of light and (4) temperature.

The specific rotation is defined as the rotation produced by a decimetre (10 cm) long column of the liquid containing 1 gram of the active substance in one cc of the solution. Therefore,

$$S_\lambda^t = \frac{10\,\theta}{l\,C}$$

where S_λ^t, represents the specific rotation at temperature t °C for a wavelength λ, θ is the angle of rotation, l is the length of the solution in cm through which the plane polarized light passes and C is the concentration of the active substance in g/cc in the solution.

The angle through which the plane of polarization is rotated by the optically active substance is determined with the help of a polarimeter. When this instrument is used to determine the quantity of sugar in a solution, it is known as a saccharimeter.

10.35 LAURENT'S HALF SHADE POLARIMETER

It consists of two nicol prisms N_1 and N_2. N_1 is a polarizer and N_2 is an analyser. Behind N_1, there is a half wave plate of quartz Q which covers one half of the field of view, while the other half G is a glass

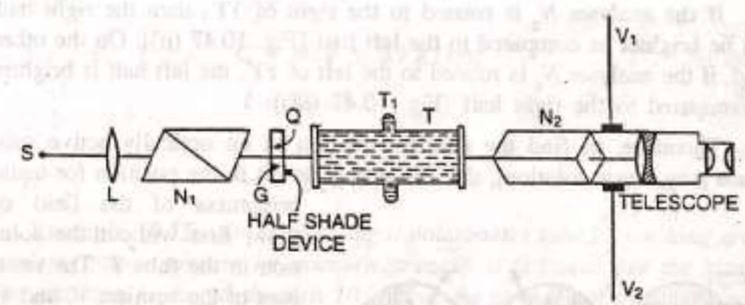


Fig. 10.45

plate. The glass plate G absorbs the same amount of light as the quartz plate Q. T is a hollow glass tube having a large diameter at its middle portion. When this tube is filled with the solution containing an optically active substance and closed at the ends by cover-slips and metal covers, there will be no air bubbles in the path of light. The air bubbles (if any) will appear at the upper portion of the wide bore T_1 of the tube.

Light from a monochromatic source S is incident on the converging lens L. After passing through N_1, the beam is plane polarized. One half of the beam passes through the quartz plate Q and the other half passes through the glass plate G. Suppose the plane of vibration of the plane polarized light incident on the half shade plate is along AB (Fig. 10.46). Here AB makes an angle θ with YY'. On passing through the quartz plate Q, the beam is split up into ordinary and extraordinary components which travel along the same direction but with different speeds and on emergence a phase difference of π or a path difference of $\frac{\lambda}{2}$ is introduced between them.

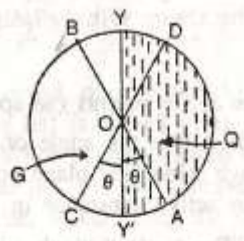


Fig. 10.46

The vibrations of the beam emerging out of quartz will be along CD whereas the vibrations of the beam emerging out of the glass plate will be along AB. If the analyser N_2 has its principal plane or section along YY' *i.e.*, along the direction which bisects the angle AOC, the amplitudes of light incident on the analyser N_2 from both the halves (*i.e.*, quartz half and glass half) will be equal. Therefore, the field of view will be equally bright [Fig. 10.47 (*i*)].

If the analyser N_2 is rotated to the right of YY', then the right half will be brighter as compared to the left half [Fig. 10.47 (*ii*)]. On the other hand, if the analyser N_2 is rotated to the left of YY', the left half is brighter as compared to the right half [Fig. 10.47 (*iii*)].

Therefore, to find the specific rotation of an optically active substance [say, sugar solution], the analyser N_2 is set in the position for equal brightness of the field of view, first without the solution in the tube T. The readings of the verniers V_1 and V_2 are noted. When a tube containing the solution of known concentration is placed, the vibrations from the quartz half and the glass half are rotated. In the case of sugar solution, AB and CD are rotated in the clockwise direction. Therefore, on the introduction of the tube containing the sugar solution, the field of view is not equally bright. The analyser is rotated in the clockwise direction and is brought to a position

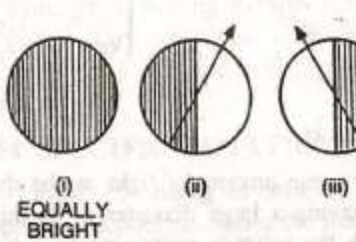


Fig. 10.47

so that the whole field of view is equally bright. The new positions of the verniers V_1 and V_2 on the circular scale are read. Thus, the angle through which the analyser has been rotated gives the angle through which the plane of vibration of the incident beam has been rotated by the sugar solution. In the actual experiment, for various concentrations of the sugar solution, the corresponding angles of rotation are determined. A graph is plotted between concentration C and the angle of rotation θ. The graph is a straight line (Fig. 10.48).

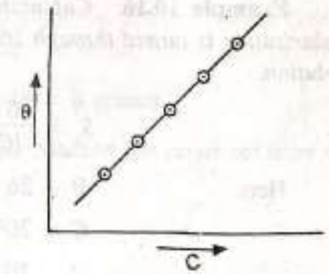


Fig. 10.48

Then from the relation

$S'_\lambda = \frac{10\,\theta}{lC}$, the specific rotation of the optically active substance is calculated.

Example 10.14. *Determine the specific rotation of the given sample of sugar solution if the plane of polarization is turned through 13.2°. The length of the tube containing 10% sugar solution is 20 cm.*

Here,

$$\theta = 13.2°$$

$$C = 10\% = 0.1\ \text{g/cm}^3$$

$$l = 20\ \text{cm}$$

$$S'_\lambda = \frac{10 \times 13.2}{20 \times 0.1} = 66°$$

Example 10.15. *On introducing a polarimeter tube 25 cm long and containing sugar solution of unknown strength, it is found that the plane of polarization is rooted through 10°. Find the strength of the sugar solution in g/cm³. (Given that the specific rotation of sugar solution is 60° per decimetre per unit concentration)* *(Rajasthan 1966)*

Here,

$$\theta = 10°$$

$$S = 60°$$

$$l = 25\ \text{cm}$$

$$S = \frac{10\,\theta}{lC}$$

$$\therefore \quad f_1 = \frac{r_1^2}{\lambda} \quad ...(i)$$

$$f_2 = \frac{r_1^2}{3\lambda} \quad ...(ii)$$

Also
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\therefore \quad \frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$$

$$\frac{1}{v_2} - \frac{1}{u} = \frac{1}{f_2}$$

Here
$$v_1 = 0.3 \text{ m} \quad \text{and} \quad v_2 = 0.6 \text{ m}$$

$$\frac{1}{0.3} - \frac{1}{u} = \frac{\lambda}{r_1^2} \quad ...(iii)$$

$$\frac{1}{0.06} - \frac{1}{u} = \frac{3\lambda}{r_1^2} \quad ...(iv)$$

Multiplying equation (*iii*) by 3 and equating with (*iv*),

$$\frac{1}{0.1} - \frac{3}{u} = \frac{1}{0.06} - \frac{1}{u}$$

$$u = \mathbf{-0.3\ m} \quad ...(v)$$

Negative sign shows that the point source is to the left of the zone plate and its distance is 0.3 m.

Substituting the value of u and λ in equation (*iii*)

$$\frac{1}{0.3} + \frac{1}{0.3} = \frac{5 \times 10^{-7}}{r_1^2}$$

$$r_1 = \mathbf{2.74 \times 10^{-4}\ m} \quad ...(vi)$$

From equation (*i*)

$$f_1 = \frac{(2.74 \times 10^{-4})^2}{5 \times 10^{-7}} = \mathbf{0.15\ m}$$

Example 9.6. *A zone plate is made by arranging the radii of the circles which define the zones such that they are the same as the radii of newton's rings formed between a plane surface and the surface having radius of curvature 200 cm. Find the principal focal length of the zone plate.* [*Delhi (Hons) 1992*]

For Newton' rings,

radius of the n th ring,

$$r_n = \sqrt{n \lambda R}$$

$$r_1 = \sqrt{\lambda R} \quad ...(i)$$

For a zone plate, the principal focal length

$$f_1 = \frac{r_1^2}{\lambda} \quad ...(ii)$$

From (*i*) and (*ii*)

$$f_1 = \frac{\lambda R}{\lambda} = R$$

But
$$R = 200 \text{ cm} = \mathbf{2\ m}$$

$$\therefore \quad f_1 = \mathbf{2\ m}$$

9.7 FRESNEL AND FRAUNHOFER DIFFRACTION

Diffraction phenomena can conveniently be divided into two groups viz. (*i*) Fresnel diffraction phenomena and (*ii*) Fraunhofer diffraction phenomena. In the Fresnel class of diffraction, the source or the screen or both are at finite distances from the aperture or obstacle causing diffraction. In this case, the effect at a specific point on the screen due to the exposed incident wavefront is considered and no modification is made by lenses and mirrors. In such a case, the phenomenon observed on the screen is called Fresnel diffraction pattern. In the Fraunhofer class of diffraction phenomena, the source and the screen on which the pattern is observed are at infinite distances from the aperture or the obstacle causing diffraction. Fraunhofer diffraction pattern can be easily observed in practice. The incoming light is rendered parallel with a lens and the diffracted beam is focussed on the screen with another lens. Observation of Fresnel diffraction phenomena do not require any lenses. Theoretical treatment of Fraunhofer diffraction phenomena is simpler. Fresnel class of diffraction phenomena are treated first in this chapter.

9.8 DIFFRACTION AT A CIRCULAR APERTURE

Let AB be a small aperture (say a pin hole) and S is a point source of monochromatic light. XY is a screen perpendicular to the plane of the paper and P is a point on the screen. SP is perpendicular to the screen. O is the centre of the aperture and r is the radius of the aperture. Let the distance of the source from the aperture be a ($SO = a$) and the distance of the screen from the aperture be b ($OP = b$). P_1OQ_1 is the incident spherical wavefront and with reference to the point P, O is the pole of the wavefront (Fig. 9.8). To consider the intensity at P, half period zones can be constructed with P as centre and radii $b+\frac{\lambda}{2}, b+\frac{2\lambda}{2}$ etc., on the exposed wavefront AOB. Depending on the distance of P from the aperture (*i.e.*, the distance b) the number of half period zones that can be constructed may be odd or even. If the distance a is such that only one half period zone can be constructed, then the intensity at P will be proportional to m_1^2 (where m_1 is the amplitude due to the first zone at P). On the other hand, if the whole of the wavefront is exposed to the point P, the resultant amplitude is $\frac{m_1}{2}$ or the intensity at P will be proportional to $\frac{m_1^2}{4}$. The position of the screen can be altered so as to construct 2, 3 or more half period zones for the same area of the aperture. If only 2 zones are exposed, the resultant amplitude at $P = m_1 - m_2$ (minimum) and if 3 zones are exposed, the amplitude $= m_1 - m_2 + m_3$ (maximum) and so on. Thus, by continuously altering the value of b, the point P becomes alternately bright and dark depending on whether odd or even number of zones are exposed by the aperture.

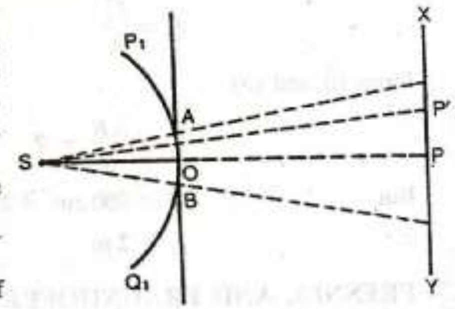


Fig 9.8

Now consider a point P' on the screen XY (Fig. 9.9) Join S to P'. The line SP' meets the wavefront at O'. O' is the pole of the wavefront

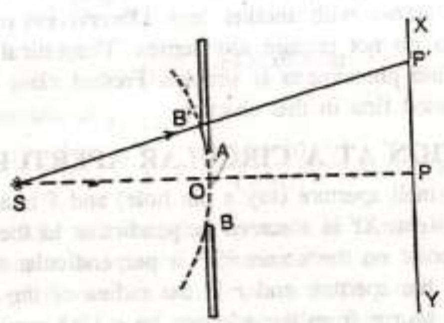


Fig. 9.9

with reference to the point P'. Construct half period zones with the point O' as the pole of the wavefront. The upper half of the wavefront is cut off by the obstacle. If the first two zones are cut off by the obstacle between the points O' and A and if only the 3 rd, 4 th and 5 th zones are exposed by the aperture AOB, then the intensity at P' will be maximum. Thus, if odd number of half period zones are exposed, point P' will be of maximum intensity and if even number of zones are exposed, the point P' will be of minimum intensity. As the distance of P' from P increases, the intensity of maxima and minima gradually decreases, because, with the point P' far removed from P, the most effective central half period zones are cut off by the obstacle between the points O' and A. With the outer zones, the obliquity increases with reference to the point P' and hence the intensity of maxima and minima also will be less. If the point P' happens to be of maximum intensity, then all the points lying on a circle of radius PP' on the screen will also be of maximum intensity. Thus, with a circular aperture, the diffraction pattern will be concentric bright and dark rings with the centre P bright or dark depending on the distance b. The width of the rings continuously decreases.

9.9 MATHEMATICAL TREATMENT OF DIFFRACTION AT A CIRCULAR APERTURE

In Fig. 9.10, S is a point source of monochromatic light, AB is the circular aperture and P is a point on the screen. O is the centre of the circular aperture. The line SOP is perpendicular to the circular aperture AB and the screen at P. The screen is perpendicular to the plane of the paper.

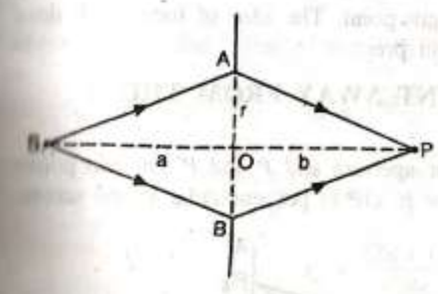


Fig. 9.10

Let δ be the path difference for the waves reaching P along the paths SAP and SOP.

$$SO = a; \quad OP = b; \quad OA = r$$

$$\delta = SA + AP - SOP$$

$$= (a^2 + r^2)^{1/2} + (b + r^2)^{1/2} - (a + b)$$

$$= a\left(1 + \frac{r^2}{a^2}\right)^{1/2} + b\left(1 + \frac{r^2}{b^2}\right)^{1/2} - (a + b)$$

$$= a\left(1+\frac{r^2}{2a^2}\right)+b\left(1+\frac{r^2}{2b^2}\right)-(a+b)$$

$$\delta = \frac{r^2}{2}\left(\frac{1}{a}+\frac{1}{b}\right)$$

$$\therefore \quad \frac{1}{a}+\frac{1}{b} = \frac{2\delta}{r^2} \qquad \text{...(i)}$$

If the position of the screen is such that n full number of half period zones can be constructed on the aperture, then the path difference

$$\delta = \frac{n\lambda}{2} \quad \text{or} \quad 2\delta = n\lambda$$

Substituting this value of 2δ in (i)

$$\frac{1}{a}+\frac{1}{b} = \frac{n\lambda}{r^2} \qquad \text{...(ii)}$$

The point P will be of maximum or minimum intensity depending on whether n is odd or even. If the source is at infinite distance (for an incident plane wavefront), then $a = \infty$ and

$$\frac{1}{b} = \frac{1}{f} = \frac{n\lambda}{r^2} \qquad \text{...(iii)}$$

If n is odd, P will be a bright point. The idea of focus at P does not mean that if is always a bright point.

9.10 INTENSITY AT A POINT AWAY FROM THE CENTRE

In Fig. 9.11, AB is a circular aperture and P and P' are two points on the screen. $PP' = x$ and $OP = b$. OP is perpendicular to the screen.

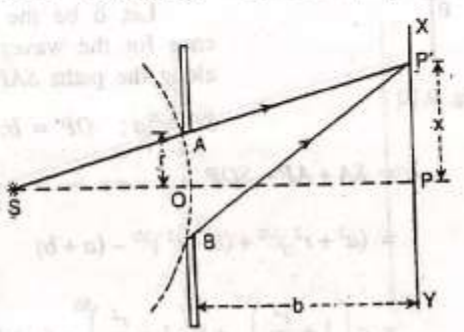


Fig. 9.11

Let r be the radius of the aperture. The path difference between the secondary waves from A and B and reaching P' can be given by

$$\delta = BP' - AP'$$

$$= \sqrt{b^2+(x+r)^2} - \sqrt{b^2+(x-r)^2}$$

$$= b\left(1+\frac{(x+r)^2}{2b^2}\right) - b\left(1+\frac{(x-r)^2}{2b^2}\right)$$

$$= b+\frac{(x+r)^2}{2b} - b - \frac{(x-r)^2}{2b}$$

$$= \frac{1}{2b}[(x+r)^2-(x-r)^2]$$

$$\delta = \frac{1}{2b}(4xr) = \frac{2rx}{b} \qquad \text{...(i)}$$

The point P' will be dark if the path difference $\delta = 2n\frac{\lambda}{2}$

(2 n means even number of zones).

$$2n\frac{\lambda}{2} = \frac{2rx_n}{2}$$

or

$$x_n = \frac{nb\lambda}{2r} \qquad \text{...(ii)}$$

where x_n gives the radius of the n th dark ring.

Similarly, if $\quad \delta = \frac{(2n+1)\lambda}{2}$,

$$\frac{(2n+1)\lambda}{2} = \frac{2rx_n}{b}$$

or

$$x_n = \frac{(2n+1)b\lambda}{4r} \qquad \text{...(iii)}$$

where x_n gives the radius of the n th bright ring.

The objective of a telescope consists of an achromatic convex lens and a circular aperture is fixed in front of the lens. Let the diameter of the aperture be D ($= 2r$). While viewing distant objects, the incident wave-front is plane and the diffraction pattern consists of a bright centre surrounded by dark and bright rings of gradually decreasing intensity. The radii of the dark rings is given by

$$x_n = \frac{nb\lambda}{2r} = \frac{nb\lambda}{D} \qquad \text{...(iv)}$$

zones are more oblique with reference to the point P. Thus (at P) the centre of the geometrical shadow will be bright as if the disc were absent. The diffraction pattern consists of a central bright spot surrounded by alternate bright and dark rings as shown in Fig. 9.12 (*b*).

9.12 DIFFRACTION PATTERN DUE TO A STRAIGHT EDGE

Let S be narrow slit illuminated by a source of monochromatic light of wavelength λ. The length of the slit is perpendicular to the plane of the paper. AD is the straight edge and the length of the edge is parallel to the length of the slit (Fig. 9.13) XY is the incident cylindrical wavefront. P is a point on the screen and SAP is perpendicular to the screen. The screen is perpendicular to the plane of the paper. Below the point P is the geometrical shadow and above P is the illuminated portion. Let the

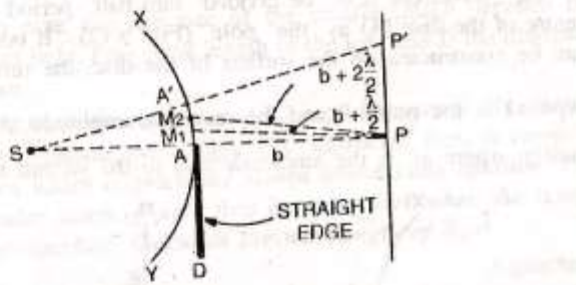


Fig. 9.13

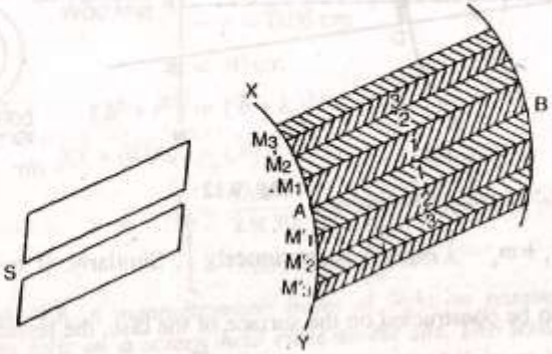


Fig. 9.14

distance AP be b. With reference to the point P, the wavefront can be divided into a number of a half period strips as shown in Fig. 9.14. XY

is the wavefront, A is the pole of the wavefront and AM_1, M_1M_2, M_2M_3 etc. measure the thickness of the 1 st, 2 nd, 3 rd etc. half period strips. With the increase in the order of the strip, the area of the strip decreases (Fig. 9.14).

In Fig. 9.13,

$$AP = b, \qquad PM_1 = b + \frac{\lambda}{2}$$

$$PM_2 = b + \frac{2\lambda}{2} \text{ etc.}$$

Let P' be a point on the screen in the illuminated portion (Fig. 9.15). To calculate the resultant effect at P' due to the wavefront XY, join S to P'. This line meets the wavefront at B. B is the pole of the wavefront with reference to the point P' and the intensity at P' will depend mainly on the number of half period strips enclosed between the points A and

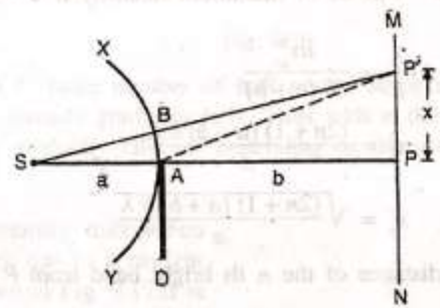


Fig. 9.15

B. The effect at P' due to the wavefront above B is the same at all points on the screen whereas it is different at different points due to the wavefront between B and A. The point P' will be of maximum intensity, if the number of half period strips enclosed between B and A is odd and the intensity at P' will be minimum if the number of half period strips enclosed between B and A is even.

9.13 POSITIONS OF MAXIMUM AND MINIMUM INTENSITY

Let the distance between the slit and the straight edge be a and the distance between the straight edge and the screen be b (Fig. 9.15). Let PP' be x.

The path difference,

$$\delta = AP' - BP'$$

able. There is no marked transition between the diffraction bands observed in the geometrical shadow and the illuminated portion. The intensity distribution on the screen due to a narrow slit (say less than the wavelength of light) a broad central maximum will be observed in the illuminated portion and the intensity variation cannot be distinguished. The intensity gradually falls off in the region of the geometrical shadow.

9.16 DIFFRACTION DUE TO NARROW WIRE

In Fig. 9.20, S is a narrow slit illuminated by monochromatic light, AB is the diameter of the narrow wire and MN is the screen. The length of the wire is parallel to the illuminated slit and perpendicular to the plane of the paper. The screen is also perpendicular to the plane of the paper. XY is the incident cylindrical wavefront and P is a point on the screen such that SOP is perpendicular to the screen. EF is the region of the geometrical shadow and above E and below F, the screen is illuminated.

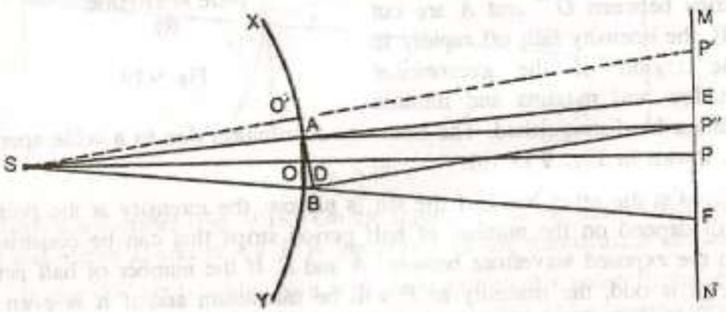


Fig. 9.20

Now consider a point P' on the screen in the illuminated portion. Join S meeting the wavefront at O'. O' is the pole of the wavefront with reference to P'. The intensity at P' due to the wavefront above O' is the same at all points and the effect due to the wavefront BY is negligible. The intensity at P' will be maximum or minimum depending on whether the number of half period strips between O' and A is odd or even. Thus, in the illuminated portion of the screen, diffraction bands of gradually diminishing intensity will be observed. The distinction between maxima and minima will become less if P' is far away from the edge E of the geometrical shadow. Maxima and minima cannot be distinguished if the wire is very narrow, because in that case the portion BY of the wavefront also produces illumination at P.

Next consider a point P'' in the region of the geometrical shadow. Interference bands of equal width will be observed in this region due to the fact the points A and B of the incident wavefront, are similar to two coherent sources. The point P'' will be of maximum or minimum intensity, depending on whether the path difference $(BP'' - AP'')$ is equal to even or odd multiples of $\lambda/2$. The fringe width β is given by

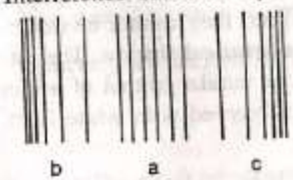


Fig 9.21

$$\beta = \frac{D\lambda}{d}$$

where D is the distance between the wire and the screen, λ is the wavelength of light and d is the distance between the two coherent sources. In this case, $d = 2r$ where $2r$ is the diameter of the wire ($AB = 2r$).

$$\therefore \quad \beta = \frac{D\lambda}{2r} \quad ...(i)$$

$$\therefore \quad r = \frac{D\lambda}{2\beta} \quad ...(ii)$$

$$\text{or} \quad \lambda = \frac{2r\beta}{D} \quad ...(iii)$$

Here, β the fringe width corresponds to the distance between any two consecutive maxima. Thus, from equations (*ii*) and (*iii*), knowing the values of r or λ; λ or r can be determined. In Fig. 9.21, the bands marked "*a*" represent the interference bands in the region of the geometrical shadow and the bands marked *b*, *b* represent the diffraction bands in the illuminated portion. The intensity distribution due to a narrow wire is shown in Fig. 9.22 (*a*). The centre of the geometrical shadow is bright.

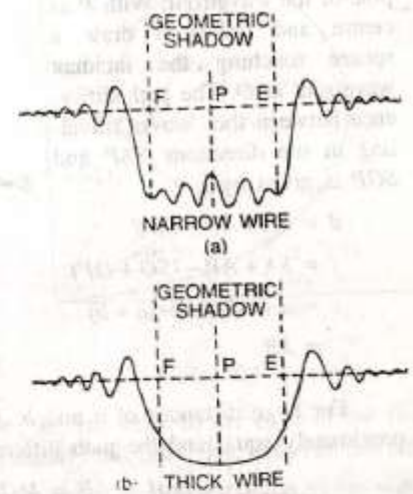


Fig 9.22

On the other hand, if the wire is very thick, the interference bands cannot be noticed.

From equation (*i*),

$$\beta = \frac{D\lambda}{2r}$$

where β is the fringe width. As the diameter of the wire increases the fringe width decreases and if the wire is sufficiently thick, the width of the interference fringes decreases considerably and they cannot be distinguished. The intensity falls off rapidly in the geometrical shadow. The diffraction pattern in the illuminated portion will be similar to that of a thin wire [Fig. 9.22 (*b*)]. Coloured fringes will be observed with white light.

9.17 CORNU'S SPIRAL

To find the effect at a point due to an incident wavefront Fresnel's method consists in dividing the wavefront into half period strips or half period zones. the path difference between the secondary waves from two corresponding points of neighbouring zones is equal to $\frac{\lambda}{2}$.

In Fig. 9.23, S is a point source of light and XY is the incident spherical wavefront. With reference to the point P, O is the pole of the wavefront. Let a and b be the distances of the points S and P from the pole of the wavefront. With P as centre and radius b draw a sphere touching the incident wavefront at O. The path difference between the waves travelling in the directions SAP and SOP is given by

$$\begin{aligned} d &= SA + AP - SOP \\ &= SA + AP - (SO + OP) \\ &= a + AB + b - (a+b) \\ &= AB \end{aligned}$$

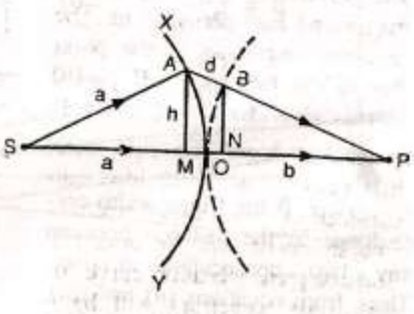


Fig. 9.23

For large distances of a and b, AM and BN can be taken to be approximately equal and the path difference d can be written as

$$d = AB = MO + ON$$

But, from the property of a circle,

$$MO = \frac{AM^2}{2\,SO} = \frac{h^2}{2\,a}$$

and $$ON = \frac{BN^2}{2\,OP} = \frac{h^2}{2\,b} \quad \text{approximately}$$

$$\therefore \quad d = \frac{h^2}{2\,a} + \frac{h^2}{2\,b} = \frac{h^2(a+b)}{2\,ab} \quad ...(i)$$

If AM happens to be the radius of the n th half period zone, then this path difference is equal to $\frac{n\lambda}{2}$ according to Fresnel's method of constructing the half period zones.

$$\therefore \quad \frac{h^2(a+b)}{2\,ab} = \frac{n\lambda}{2} \quad ...(ii)$$

The resultant amplitude at an external point due to the wavefront can be obtained by the following method. Let the first half period strip of the Fresnel's zones be divided into eight substrips and these vectors are represented from O to M_1 (Fig. 9.24). The continuous phase change is due to the continuous increase in the obliquity factor from O to M_1. The resultant amplitude at the external point due to the first half period strip is given by OM_1 ($= m_1$). Similarly if the process is continued, we obtain the vibration curve M_1M_2. The portion M_1M_2 corresponds to the second half period strip. The resultant amplitude at the point due to the first two half period strip is given by OM_2 ($= A$). If instead of eight substrips, each half period zone is divided into substrips of infinitesimal width, a smooth curve will be obtained. The complete vibration curve for the whole wavefront will be a spiral as shown in Fig. 9.23. X and Y correspond to the two extremities of the wavefront and M_1, M_2 etc. refer to the edge of the first, second, etc. half period strips. Similarly M_1', M_2' etc. refer to the edge of the first, second etc. half period strips of the lower portion of the wavefront. this is called Cornu's spiral. The characteristic of this curve is that for any point P on the curve, the phase lag δ is directly proportional to the square of the distance v. The distance is measured along the length of the curve from the point O. For a path difference of λ the phase difference is 2π. Hence, for a path difference of d, the phase difference δ is given by

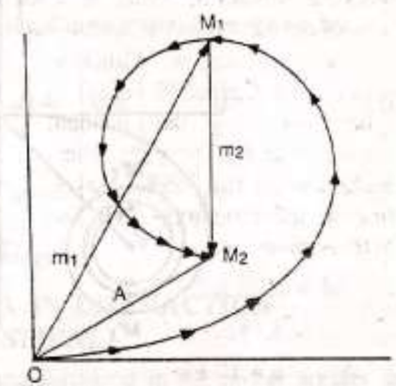


Fig. 9.24

$$\delta = \frac{2\pi}{\lambda} \cdot d$$

Substituting the value of d from equation (*ii*) in the region of the geometrical shadow. The intensity distribution due to Fresnel's diffraction at a straight edge is given in Fig. 9.17 on page 429.

9.22 FRAUNHOFER DIFFRACTION AT A SINGLE SLIT

To obtain a Fraunhofer diffraction pattern, the incident wavefront must be plane and the diffracted light is collected on the screen with the help of a lens. Thus, the source of light should either be at a large distance from the slit or a collimating lens must be used.

In Fig. 9.33, S is a narrow slit perpendicular to the plane of the paper and illuminated by monochromatic light. L_1 is the collimating lens and AB is a slit of width a. XY is the incident spherical wavefront. The light passing through the slit AB is incident on the lens L_2 and the final refracted beam is observed on the screen MN. The screen is perpendicular to the

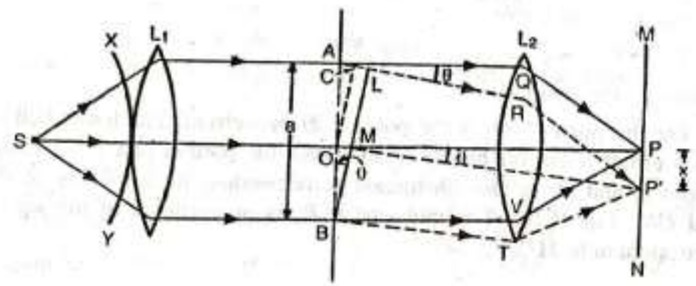


Fig. 9.33

plane of the paper. The line SP is perpendicular to the screen. L_1 and L_2 are achromatic lenses.

A plane wavefront is incident on the slit AB and each point on this wavefront is a source of secondary disturbance. The secondary waves travelling in the direction parallel to OP *viz.* AQ and BV come to focus at P and a bright central image is observed. The secondary waves from points equidistant from O and situated in the upper and lower halves OA and OB of the waterfront travel the same distance in reaching P and hence the path difference is zero. The secondary waves reinforce one another and P will be a point of maximum intensity.

Now, consider the secondary waves travelling in the direction AR, inclined at an angle θ to the direction OP. All the secondary wave travelling in this direction reach the point P' on the screen. The point P' will be of maximum or minimum intensity depending on the path difference between the secondary waves originating from the corresponding points of the wavefront. Draw OC and BL perpendicular to AR.

Then, in the ΔABL

$$\sin\theta = \frac{AL}{AB} = \frac{AL}{a}$$

or

$$AL = a\sin\theta$$

where a is the width of the slit and AL is the path difference between the secondary waves originating from A and B. If this path difference is equal to λ the wavelength of light used, then P' will be a point of minimum intensity. The whole wavefront can be considered to be of two halves OA and OB and if the path difference between the secondary waves from A and B is λ, then the path difference between the secondary waves from A and O will be $\frac{\lambda}{2}$. Similarly for every point in the upper half OA, there is a corresponding point in the lower half OB, and the path difference between the secondary waves from these points is $\frac{\lambda}{2}$. Thus, destructive interference takes place and the point P' will be of minimum intensity. If the direction of the secondary waves is such that $AL = 2\lambda$, then also the point where they meet the screen will be of minimum intensity. This is so, because the secondary waves from the corresponding points of the lower half, differ in path by $\frac{\lambda}{2}$ and this again gives the position of minimum intensity. In general

$$a\sin\theta_n = n\lambda$$

$$\sin\theta_n = \frac{n\lambda}{a}$$

where θ_n gives the direction of the n th minimum. Here n is an integer. If, however, the path difference is odd multiples of $\frac{\lambda}{2}$, the directions of the secondary maxima can be obtained. In this case,

$$a\sin\theta_n = (2n+1)\frac{\lambda}{2}$$

or

$$\sin\theta_n = \frac{(2n+1)\lambda}{2a}$$

where

$$n = 1, 2, 3 \text{ etc.}$$

Thus, the diffraction pattern due to a single slit consists of a central bright maximum at P followed by secondary maxima and minima on both the sides. The intensity distribution on the screen is given in Fig. 9.34.

P corresponds to the position of the central bright maximum and the points on the screen for which the path difference between the points A and B

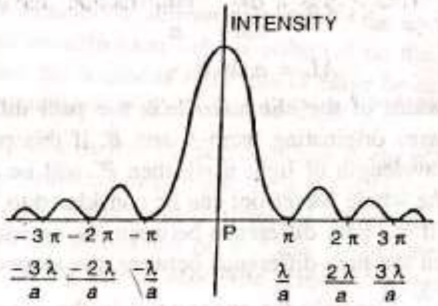


Fig. 9.34

is λ, 2λ etc., correspond to the positions of secondary minima. The secondary maxima are of much less intensity. The intensity falls off rapidly from the point P outwards.

If the lens L_2 is very near the slit or the screen is far away from the lens L_2, then

$$\sin\theta = \frac{x}{f} \qquad ...(i)$$

where f is the focal length of the lens L_2

But, $$\sin\theta = \frac{\lambda}{a} \qquad ...(ii)$$

$\therefore$ $$\frac{x}{f} = \frac{\lambda}{a}$$

or $$x = \frac{f\lambda}{a}$$

where x is the distance of the secondary minimum from the point P. Thus, the width of the central maximum $= 2x$.

or $$2x = \frac{2f\lambda}{a} \qquad ...(iii)$$

The width of the central maximum is proportional to λ, the wavelength of light. With red light (longer wavelength), the width of the central maximum is more than with violet light (shorter wavelength). With a narrow slit, the width of the central maximum is more. The diffraction pattern consists of alternate bright and dark bands with monochromatic light. With white light, the central maximum is white and the rest of the diffraction

bands are coloured. From equation (*ii*), if the width a of the slit is large, $\sin\theta$ is small and hence θ is small. The maxima and minima are very close to the central maximum at P. But with a narrow slit, a is small and hence θ is large. This results a distinct diffraction maxima and minima on both the sides of P.

Example 9.9. *Find the half angular width of the central bright maximum in the Fraunhofer diffraction pattern of a slit of width 12×10^{-5} cm when the slit is illuminated by monochromatic light of wavelength 6000 Å.*

Here $$\sin\theta = \frac{\lambda}{a}$$

where θ is half angular width of the central maximum.

$$a = 12 \times 10^{-5}\text{ cm},\ \lambda = 6000\text{ Å} = 6 \times 10^{-5}\text{ cm}.$$

$\therefore$ $$\sin\theta = \frac{\lambda}{a} = \frac{6 \times 10^{-5}}{12 \times 10^{-5}} = 0.50$$

or $$\theta = 30^{\circ}$$

Example 9.10. *In Fraunhofer diffraction due to a narrow slit a screen is placed 2* m *away from the lens to obtain the pattern. If the slit width is 0.2* mm *and the first minima lie 5* mm *on either side of the central maximum, find the wavelength of light.* *[Delhi (Sub) 1977]*

In the case of Fraunhofer diffraction at a narrow rectangular aperture,

$$a\sin\theta = n\lambda$$

$$n = 1$$

$\therefore$ $$a\sin\theta = \lambda$$

$$\sin\theta = \frac{x}{D}$$

$\therefore$ $$\frac{ax}{D} = \lambda$$

$$\lambda = \frac{ax}{D}$$

Here $$a = 0.2\text{ mm} = 0.02\text{ cm}$$

$$x = 5\text{ mm} = 0.5\text{ cm}$$

$$D = 2\text{ m} = 200\text{ cm}$$

$\therefore$ $$\lambda = \frac{0.02 \times 0.5}{200}$$

$$\lambda = 5 \times 10^{-5}\text{ cm}$$

$$\lambda = \mathbf{5000\text{ Å}}$$

of the aperture and P is a point on the screen. CP is perpendicular to the screen. The screen is perpendicular to the plane of the paper. A plane wavefront is incident on the circular aperture. The secondary waves travelling in the direction CO come to focus at P. Therefore, P corresponds to the position of the central maximum. Here, all the secondary waves emanating from points equidistant from O travel the same distance before reaching P and hence they all reinforce one another. Now consider the secondary waves travelling in a direction inclined at an angle θ with the direction CP. All these secondary waves meet at P_1 on the screen. Let the distance PP_1 be x. The path difference between the secondary waves emanating from the points B and A (extremities of a diameter) is AD.

From the Δ ABD,

$$AD = d \sin \theta$$

Arguing as in Article 9.22, the point P_1 will be of minimum intensity if this path difference is equal to integral multiples of λ *i.e.*,

$$d \sin \theta_n = n\lambda \qquad ...(i)$$

The point P_1 will be of maximum intensity if the path difference is equal to odd multiples of $\frac{\lambda}{2}$ *i.e.*,

$$d \sin \theta_n = \frac{(2n+1)\lambda}{2} \qquad ...(ii)$$

If P_1 is a point of minimum intensity, then all the points at the same distance from P as P_1 and lying on a circle of radius x will be of minimum intensity. Thus, the diffraction pattern due to a circular aperture consists of a central bright disc called the Airy's disc, surrounded by alternate dark and bright concentric rings called the Airy's rings. The intensity of the dark rings is zero and that of the bright rings decreases gradually outwards from P.

Further, if the collecting lens is very near the slit or when the screen is at a large distance from the lens,

$$\sin \theta = \theta = \frac{x}{f} \qquad ...(iii)$$

Also, for the first secondary minimum,

$$d \sin \theta = \lambda$$

$$\sin \theta = \theta = \frac{\lambda}{d} \qquad ...(iv)$$

From equations (*iii*) and (*iv*)

$$\frac{x}{f} = \frac{\lambda}{d}$$

or

$$x = \frac{f\lambda}{d} \qquad ...(v)$$

where x is the radius of the Airy's disc. But actually, the radius of the first dark ring is slightly more than that given by equation (*v*). According to Airy, it is given by

$$x = \frac{1.22 f\lambda}{d} \qquad ...(vi)$$

The discussion of the intensity distribution of the bright and dark rings is similar to the one given for a rectangular slit. With increase in the diameter of the aperture, the radius of the central bright ring decreases.

Example 9.16. *In Fraunhofer diffraction pattern due to a single slit, the screen is at a distance of 100* cm *from the slit and the slit is illuminated by monochromatic light of wavelength 5893* Å. *The width of the slit is 0.1* mm. *Calculate the separation between the central maximum and the first secondary minimum.* ***(Mysore)***

For a rectangular slit,

$$x = \frac{f\lambda}{d}$$

Here $f = 100$ cm, $\lambda = 5893$ Å

$$= 5893 \times 10^{-8} \text{ cm},$$

$$d = 0.1 \text{ mm} = 0.01 \text{ cm}, x = ?$$

$$\therefore \quad x = \frac{100 \times 5893 \times 10^{-8}}{0.01} = \mathbf{0.5893\ cm}$$

9.26 FRAUNHOFER DIFFRACTION AT DOUBLE SLIT

In Fig. 9.40, AB and CD are two rectangular slits parallel to one another and perpendicular to the plane of the paper. The width of each slit is a and the width of the opaque portion is b. L is a collecting lens and MN is a screen perpendicular to the plane of the paper. P is a point on the screen such that OP is perpendicular to the screen. Let a plane wavefront be incident on the surface of XY. All the secondary waves travelling in a direction parallel to OP come to focus at P. Therefore, P corresponds to the position of the central bright maximum.

In this case, the diffraction pattern has to be considered in two parts : (*i*) the interference phenomenon due to the secondary waves emanating from th corresponding points of the two slits and (*ii*) the diffraction pattern due to the secondary waves from the two slits individually. For calculating

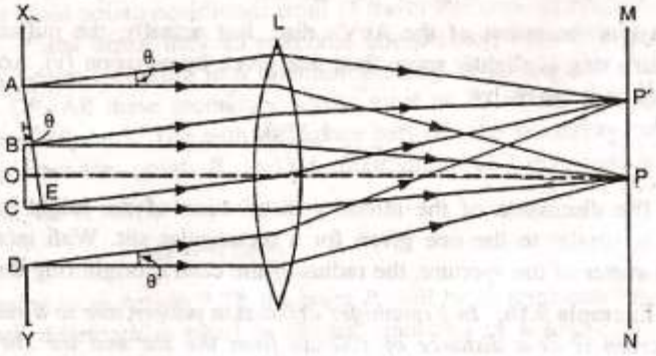


Fig. 9.40

the positions of interference maxima and minima, the diffracting angle is denoted as θ and for the diffraction maxima and minima it is denoted as ϕ. Both the angles θ and ϕ refer to the angle between the direction of the secondary waves and the initial direction of the incident light.

(*i*) **Interference maxima and minima.** Consider the secondary waves travelling in a direction inclined at an angle θ with the initial direction.

In the Δ *ACN* (Fig. 9.41)

$$\sin\theta = \frac{CN}{AC} = \frac{CN}{a+b}$$

or $$CN = (a+b)\sin\theta$$

If this path difference is equal to odd multiples of $\frac{\lambda}{2}$, θ gives the direction of minima due to interference of the secondary waves from the two slits.

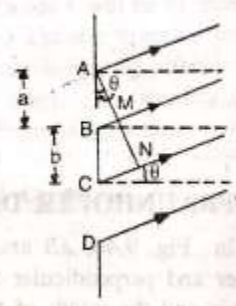


Fig. 9.41

$$\therefore \quad CN = (a+b)\sin\theta_n = (2n+1)\frac{\lambda}{2} \quad \ldots(i)$$

Putting n = 1, 2, 3 etc., the values of $\theta_1, \theta_2, \theta_3$ etc., corresponding to the directions of minima can be obtained.

From equation (*i*)

$$\sin\theta_n = \frac{(2n+1)\lambda}{2(a+b)} \quad \ldots(ii)$$

On the other hand, if the secondary waves travel in a direction θ′ such that the path difference is even multiples of $\frac{\lambda}{2}$, then θ′ gives the direction of the maxima due to interference of light waves emanating from the two slits.

$$\therefore \quad CN = (a+b)\sin\theta'_n = 2n.\frac{\lambda}{2}$$

or $$\sin\theta'_n = \frac{n\lambda}{(a+b)} \quad \ldots(iii)$$

Putting n = 1, 2, 3 etc., the values $\theta'_1, \theta'_2, \theta'_3$ etc., corresponding to the directions of the maxima can be obtained.

From equation (*ii*)

$$\sin\theta_1 = \frac{3\lambda}{2(a+b)}$$

and $$\sin\theta_2 = \frac{5\lambda}{2(a+b)}$$

$$\therefore \quad \sin\theta_2 - \sin\theta_1 = \frac{\lambda}{a+b} \quad \ldots(iv)$$

Thus, the angular separation between any two consecutive minima (or maxima) is equal to $\frac{\lambda}{a+b}$. The angular separation is inversely proportional to $(a + b)$, the distance between the two slits.

(*ii*) **Diffraction maxima and minima.** Consider the secondary waves travelling in a direction inclined at an angle ϕ with the initial direction of the incident light.

If the path difference *BM* is equal to λ the wavelength of light used, then ϕ will give the direction of diffraction minimum (Fig. 9.41). That is, the path difference between the secondary waves emanating from the extremities of a slit (*i.e.*, points *A* and *B*) is equal to λ. Considering the wavefront on *AB* to be made up of two halves, the path difference between the corresponding points of the upper and the lower halves is equal to $\frac{\lambda}{2}$. The effect at *P*′ due to the wavefront incident on *AB* is zero. Similarly for the same direction of the secondary waves, the effect at P' due to the wavefront incident on th slit CD is also zero. In general,

$$a \sin \phi_n = n\lambda \qquad ...(v)$$

Putting $n = 1, 2$ etc., the values of ϕ_1, ϕ_2 etc., corresponding to the directions of diffraction minima can be obtained.

9.27 FRAUNHOFER DIFFRACTION AT DOUBLE SLIT CALCULUS METHOD)

The intensity distribution due to Fraunhofer diffraction at double slit two parallel slits) can be obtained by integrating the expression for dy vide single slit) for both the slits.

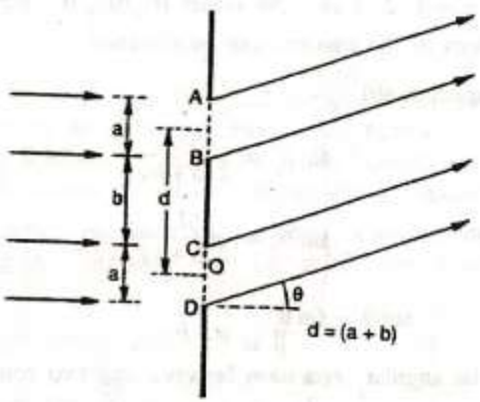


Fig. 9.42

Here

$$y = K\left[\int_{-\frac{a}{2}}^{+\frac{a}{2}} \sin\left[2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{z\sin\theta}{\lambda}\right)\right]dz + \int_{d-\frac{a}{2}}^{d+\frac{a}{2}} \sin\left[2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{z\sin\theta}{\lambda}\right)\right]dz\right]$$

$$\therefore \quad y = Ka\left(\frac{\sin\alpha}{\alpha}\right)\left[\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right)\right]$$

$$-\frac{K\lambda}{2\pi\sin\theta}\left[\cos 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{z\sin\theta}{\lambda}\right)\right]_{d-\frac{a}{2}}^{d+\frac{a}{2}}$$

$$y = Ka\left(\frac{\sin\alpha}{\alpha}\right)\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right)$$

$$-\frac{K\lambda}{2\pi\sin\theta}\left[\cos 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}+\frac{a\sin\theta}{2\lambda}\right)\right.$$

$$\left.-\cos 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}-\frac{a\sin\theta}{2\lambda}\right)\right]$$

$$y = Ka\left(\frac{\sin\alpha}{\alpha}\right)\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right)$$

$$+\frac{K\lambda}{\pi\sin\theta}\left[\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}\right)\sin\left(\frac{\pi a\sin\theta}{\lambda}\right)\right]$$

But $\quad \alpha = \dfrac{\pi a\sin\theta}{\lambda}$

$$\therefore \quad y = Ka\left(\frac{\sin\alpha}{\alpha}\right)\left[\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right)\right.$$

$$\left.+\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}\right)\right]$$

$$y = 2\,Ka\left(\frac{\sin\alpha}{\alpha}\right)\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{2\lambda}\right)\cos\frac{\pi d\sin\theta}{\lambda}$$

Let $\quad \dfrac{\pi d\sin\theta}{\lambda} = \beta$

$$\therefore \quad y = 2Ka\left(\frac{\sin\alpha}{\alpha}\right)\cos\beta\,\sin 2\pi\left[\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{2\lambda}\right]$$

The intensity at a point P' is given by

$$I = 4K^2a^2\left(\frac{\sin^2\alpha}{\alpha^2}\right)\cos^2\beta$$

$$\therefore \quad I_0 = K^2a^2$$

$$\therefore \quad I = 4I_0\left(\frac{\sin^2\alpha}{\alpha^2}\right)\cos^2\beta$$

The intensity of the central maximum $= 4I_0$ when $\alpha = 0$ and $\beta = 0$.

In Fig. 9.43, the dotted curve represents the intensity distribution due to diffraction pattern due to double slit and the thick line curve represents the intensity distribution due to interference between the light from both the slits. The pattern consists of diffraction maxima within each diffraction maximum.

The intensity distribution due to Fraunhofer diffraction at two parallel slits is shown in Fig. 9.43. The full line represents equally spaced interference maxima and minima and the dotted curve represents the diffraction maxima and minima. In the region originally occupied by the

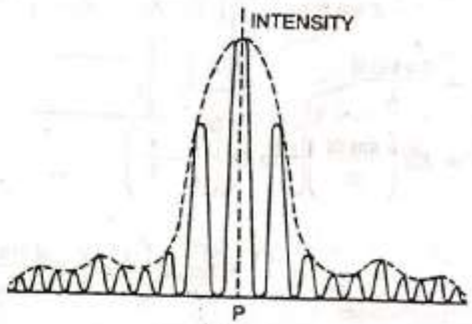


Fig. 9.43

central maximum of the single slit diffraction pattern, equally spaced interference maxima and minima are observed . The intensity of the central interference maximum is four times the intensity of the central maximum of the single slit diffraction pattern. The intensity of the other interference maxima on the two sides of the central maximum of the single slit diffraction pattern. The intensity of the other interference maxima on the two sides of the central maximum gradually decreases. In the region of the secondary maxima due to diffraction at a single slit, equally spaced interference maxima of low intensity are observed. The intensity

distribution shown in Fig. 9.43 corresponds to $2a = b$ where a is the width of each slit and b is the opaque spacing between the two slits. Thus, the pattern due to diffraction at a double slit consists of a diffraction pattern due to the individual slits of width a each and the interference maxima and minima of equal spacing. The spacing of the interference maxima and minima is dependent on the values of a and b.

9.28 DISTINCTION BETWEEN SINGLE SLIT AND DOUBLE SLIT DIFFRACTION PATTERNS

The single slit diffraction pattern consists of a central bright maximum with secondary maxima and minima of gradually decreasing intensity. The double slit diffraction pattern consists of equally spaced interference maxima and minima within the central maximum. The intensity of the central maximum in the diffraction pattern due to a double slit is four times that of the central maximum due to diffraction at a single slit. In the above arrangement, if one of the slits is covered with an opaque screen, the pattern observed is similar to the one observed with a single slit. The spacing of the diffraction maxima and minima depends on a, the width of the slit and the spacing of the interference maxima and minima depends on the value of a and b where b is the opaque spacing between the two slits. The intensities of the interference maxima are not constant but decrease to zero on either side of the central maximum. These maxima reappear two or three times before the intensity becomes too low to be observed.

9.29 MISSING ORDERS IN A DOUBLE SLIT DIFFRACTION PATTERN

In the diffraction pattern due to a double slit discussed earlier, the slit width is taken as a and the separation between the slits as b. If the slit width a is kept constant, the diffraction pattern remains the same. Keeping a constant, if the spacing b is altered the spacing between the interference maxima changes. Depending on the relative values of a and b certain orders of interference maxima will be missing in the resultant pattern.

The directions of interference maxima are given by the equation

$$(a+b)\sin\theta = n\lambda \qquad ...(i)$$

The direction of diffraction minima are given by the equation,

$$a\sin\theta = p\lambda \qquad ...(ii)$$

In equations (i) and (ii) n and p are integers. If the values of a and b are such that both the equations are satisfied simultaneously for the same values of θ, then the positions of certain interference maxima correspond to the diffraction minima at the same position on the screen.

(*i*) Let $a = b$

Then $2a \sin\theta = n\lambda$

and $a \sin\theta = p\lambda$

$\therefore \quad \frac{n}{p} = 2$

or $n = 2p$

If $p = 1, 2, 3$ etc.,

then $n = 2, 4, 6$ etc.

Thus, the orders 2, 4, 6 etc. of the interference maxima will be missing in the diffraction pattern. There will be three interference maxima in the central diffraction maximum

(*ii*) If $2a = b$

then $3a \sin\theta = n\lambda$

and $a \sin\theta = p\lambda$

$\therefore \quad \frac{n}{p} = 3$

or $n = 3p$

If $p = 1, 2, 3$ etc.

$n = 3, 6, 9$ etc.

Thus the orders 3, 6, 9 etc. of the interference maxima will be missing in the diffraction pattern. On both sides of the central maximum, the number of interference maxima is 2 and hence there will be five interference maxima in the central diffraction maximum. The position of the third interference maximum also corresponds to the first diffraction minimum.

(*iii*) If $a + b = a$

i.e., if $b = 0$

The two slits join and all the orders of the interference maxima will be missing. The diffraction pattern observed on the screen is similar to that due to a single slit of width equal to $2a$.

Example 9.17. *Deduce the missing orders for a double slit Fraunhofer diffraction pattern, if the slit widths are 0.16* mm *and they are 0.8* mm *apart.* [*Berhampur (Hons.)*]

The direction of interference maxima are given by the equation,

$$(a+b)\sin\theta = n\lambda \qquad ...(i)$$

The directions of diffraction minima are given by

$$a \sin\theta = p\lambda \qquad ...(ii)$$

$$\therefore \quad \frac{(a+b)}{a} = \frac{n}{p}$$

Here $a = 0.16 \text{ mm} = 0.016 \text{ cm}$

$b = 0.8 \text{ mm} = 0.080 \text{ cm}$

$$\therefore \quad \frac{0.016 + 0.080}{0.016} = \frac{n}{p}$$

$$\therefore \quad \frac{n}{p} = 6$$

$n = 6p$

For values of $p = 1, 2, 3$ etc.

$n = 6, 12, 18$ etc.

Thus the orders 6, 12, 18 etc. of the interference maxima will be missing in the diffraction pattern.

Example 9.18. *A diffraction phenomenon is observed using a double slit (illuminated with light of wavelength 5000 Å. The slit width is 0.02* mm *and spacing between the two slits is 0.10* mm. *The distance of the screen from the slits where the observation is made is 100* cm. *Calculate (i) the distance between the central maximum and the first minimum of the fringe envelope and (ii) the distance between any two consecutive double slit dark fringes.* [*IAS.*]

Here $a = 0.02 \text{ mm} = 2 \times 10^{-5} \text{ m}$

$b = 0.1 \text{ mm} = 10^{-4} \text{ m}$

$(a+b) = 1.2 \times 10^{-4} \text{ m}$

$\lambda = 5000 \text{ Å} = 5 \times 10^{-7} \text{ m}$

$d = 100 \text{ cm} = 1 \text{ m}$

(*i*) The angular separation between the central maximum and the first minimum is

$$\sin\theta_1 = \theta_1 = \frac{\lambda}{2(a+b)}$$

and $\theta_1 = \frac{x_1}{D}$

Thus, the resultant amplitude is proportional to N and resultant intensity is proportional to N^2.

$$\therefore \qquad I = N^2 I_0 \left(\frac{\sin^2 \alpha}{\alpha^2} \right)$$

These maxima are intense and are called principal maxima.

9.33 PLANE DIFFRACTION GRATING

A diffraction grating is an extremely useful device and in one of its forms it consists of a very large number of narrow slits side by side. The slits are separated by opaque spaces. When a wavefront is incident on a grating surface, light is transmitted through the slits and obstructed by the opaque portions. Such a grating is called a transmission grating. The secondary waves from the positions of the slits interfere with one another, similar to the interference of waves in Young's experiment. Joseph Fraunhofer used the first gratings which consisted of a large number of parallel fine wires stretched on a frame. Now, gratings are prepared by ruling equidistant parallel lines on a glass surface. The lines are drawn with a fine diamond point. The space in between any two lines is transparent to light and the lined portion is opaque to light. Such surfaces act as transmission gratings. If, on the other hand, the lines are drawn on a silvered surface (plane or concave) then light is reflected from the positions of the mirror in between any two lines and such surfaces act as reflection gratings.

If the spacing between the lines is of the order of the wave length of light, then an appreciable deviation of the light is produced. Gratings used for the study of the visible region of the spectrum contain 10,000 lines per cm. Gratings, with originally ruled surfaces are only few. For practical purposes, replicas of the original grating are prepared. On the original grating surface a thin layer of collodion solution is poured and the solution is allowed to harden. Then, the film of collodion is removed from the grating surface and then fixed between two glass plates. This serves as a plane transmission grating. A large number of replicas are prepared in this way from a single original ruled surface.

9.34 THEORY OF THE PLANE TRANSMISSION GRATING

In Fig. 9.44, XY is the grating surface and MN is the screen, both perpendicular to the plane of the paper. The slits are all parallel to one another and perpendicular to the plane of the paper. Here AB is a slit and BC is an opaque portion. The width of each slit is a and the opaque spacing between any two consecutive slits is b. Let a plane wavefront be incident on the grating surface. Then all the secondary waves travelling in the same direction as that of the incident light will come to focus at the point P on the screen. The screen is placed at the focal plane of the collecting lens. The point P where all the secondary waves reinforce one another corresponds to the position of the central bright maximum.

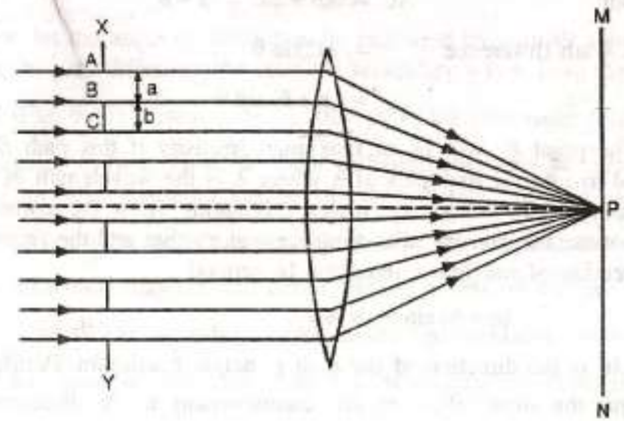


Fig. 9.44

Now, consider the secondary waves travelling in a direction inclined at an angle θ with the direction of the incident light (Fig. 9.45). The collecting lens also is suitably rotated such that the axis of the lens is

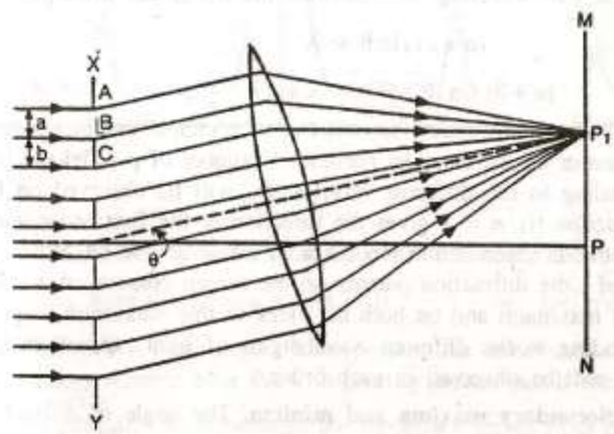


Fig. 9.45

parallel to the direction of the secondary waves. These secondary waves come to focus at the point P_1 on the screen. The intensity at P_1 will depend on the path difference between the secondary waves originating from the corresponding points A and C of two neighbouring slits. In Fig. 9.45, $AB = a$ and $BC = b$. The path difference between the secondary waves starting from A and C is equal to $AC \sin\theta$. (This will be clear from Fig. 9.41).

$$\text{But} \quad AC = AB + BC = a + b$$

$$\therefore \text{ Path difference } = AC \sin\theta$$

$$= (a+b)\sin\theta$$

The point P_1 will be of maximum intensity if this path difference is equal to integral multiples of λ where λ is the wavelength of light. In this case, all the secondary waves originating from the corresponding points of the neighouring slits reinforce one another and the angle θ gives the direction of maximum intensity. In general

$$(a+b)\sin\theta_n = n\lambda \qquad ...(i)$$

where θ_n is the direction of the n th principal maximum. Putting $n = 1, 2, 3$ etc., the angles $\theta_1, \theta_2, \theta_3$ etc. corresponding to the directions of the principal maxima can be obtained.

If the incident light consists of more than one wavelength, the beam gets dispersed and the angles of diffraction for different wavelengths will be different. Let λ and $\lambda + d\lambda$ be two nearby wavelengths present in the incident light and θ and $(\theta + d\theta)$ be the angles of diffraction corresponding to these two wavelengths. Then, for the first order principal maxima

$$(a+b)\sin\theta = \lambda$$

$$\text{and} \quad (a+b)\sin(\theta + d\theta) = \lambda + d\lambda$$

Thus, is any order, the number of principal maxima corresponds to the number of wavelengths present. A number of parallel slit images corresponding to the different wavelengths will be observed on the screen. In equation (*i*), $n = 1$ gives the direction of the first order image, $n = 2$ gives the direction of the second order image and so on. When white light is used , the diffraction pattern on the screen consists of a white central bright maximum and on both the sides of this maximum a spectrum corresponding to the different wavelengths of light present in the incident beam will be observed in each order.

Secondary maxima and minima. The angle of diffracting θ_n corresponding to the direction of the n th principal maximum is given by the equation

$$(a+b)\sin\theta_n = n\lambda$$

In this equation, $(a + b)$ is called the **grating element**. Here a is the width of the slit and b is the width of the opaque portion. For a grating with 15,000 lines per inch the value of

$$(a+b) = \frac{2.54}{15000} \text{ cm}$$

Now, let the angle of diffraction be increased by a small amount $d\theta$ such that the path difference between the secondary waves from the points A and C (Fig. 9.45) increases by $\frac{\lambda}{N}$. Here N is the total number of lines on the grating surface. Then, the path difference between the secondary waves from the extreme points of the grating surface will be $\frac{\lambda}{N} N = \lambda$. Assuming the whole wavefront to be divided into two halves, the path difference between the corresponding points of the two halves will be $\frac{\lambda}{2}$ and all the secondary waves cancel one another's effect. Thus, $(\theta_n + d\theta)$ will give the direction of the first secondary minimum after the

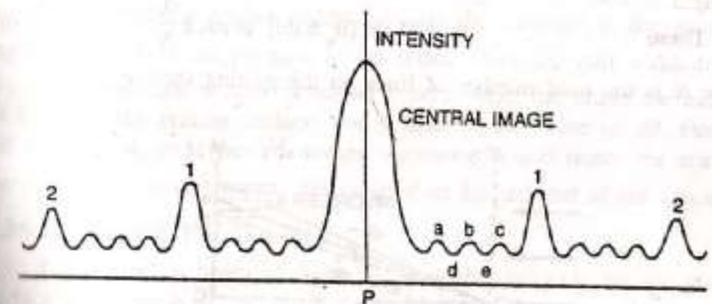


Fig. 9.46

n th primary maximum. Similarly, if the path difference between the secondary waves from the points A and C is $\frac{2\lambda}{N}, \frac{3\lambda}{N}$ etc. for gradually increasing values of $d\theta$, these angles correspond to the directions of 2 nd, 3 rd etc. secondary minima after the n th primary maximum. If the value is $\frac{2\lambda}{N}$, then the path difference between the secondary waves from the extreme points of the grating surface is $\frac{2\lambda}{N} \times N = 2\lambda$ and considering the wavefront to be divided into 4 portions, the concept of the 2 nd secondary minimum can be understood. The number of secondary minima in between any two primary maxima is $N - 1$ and the number of secondary maxima is $N - 2$.

The intensity distribution of the screen is shown in Fig. 9.46. P corresponds to the position of the central maxima and 1, 2 etc. on the two sides of P represent the 1st, 2nd etc. principal maxima. a, b, c etc. are secondary maxima and d, e etc. are the secondary minima. The intensity as well as the angular spacing of the secondary maxima and minima are so small in comparison to the principal maxima that they cannot be observed. It results in uniform darkness between any two principal maxima.

9.35 WIDTH OF PRINCIPAL MAXIMA

The direction of the n th principal maximum is given by

$$(a+b)\sin\theta_n = n\lambda \quad ...(i)$$

Let $\theta_n + d\theta$ and $\theta_n - d\theta$ give the directions of the first secondary minima on the two sides of the n th primary maxima (Fig. 9.47).

Then. $$(a+b)\sin[\theta_n \pm d\theta] = n\lambda \pm \frac{\lambda}{N} \quad ...(ii)$$

where N is the total number of lines on the grating surface.

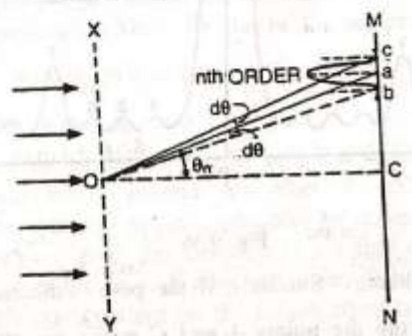


Fig. 9.47

Dividing (ii) by (i)

$$\frac{(a+b)\sin(\theta_n \pm d\theta)}{(a+b)\sin\theta_n} = \frac{n\lambda \pm \frac{\lambda}{N}}{n\lambda}$$

$$\frac{\sin(\theta_n \pm d\theta)}{\sin\theta_n} = 1 \pm \frac{1}{Nn}$$

Expanding this equation

$$\frac{\sin\theta_n \cdot \cos d\theta \pm \cos\theta_n \sin d\theta}{\sin\theta_n} = 1 \pm \frac{1}{Nn} \quad ...(iii)$$

For small values of $d\theta$; $\cos d\theta = 1$ and $\sin d\theta = d\theta$.

$$\therefore \quad 1 \pm \cot\theta_n\, d\theta = 1 \pm \frac{1}{Nn} \quad ...(iv)$$

$$\cot\theta_n\, d\theta = \frac{1}{Nn}$$

$$d\theta = \frac{1}{Nn\cot\theta_n}$$

In equation (iv), $d\theta$ refers to half the angular width of the principal maximum. The half width $d\theta$ is (i) inversely proportional to N the total number of lines and (ii) inversely proportional to $n\cot\theta_n$. The value of $n\cot\theta_n$ is more for higher orders because the increase in the value of $\cot\theta_n$ is less than the increase in the order. Thus, the half width of the principal maximum is less for higher orders. Also, the larger the number of lines on the grating surface, the smaller is the value of $d\theta$. Further, the value of θ_n is higher for longer wavelengths and hence the spectral lines are more sharp towards the violet than the red end of the spectrum.

9.36 OBLIQUE INCIDENCE

Let a parallel beam of light be incident obliquely on the grating surface at an angle of incidence i (Fig. 9.48).

Path difference between the secondary waves passing through the points A and $C = FC + CE$.

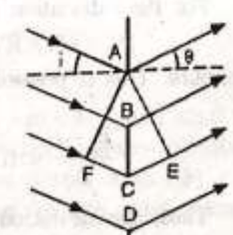
Here, $AB = a$, the width of the slit and $BC = b$, the width of the opaque portion.

From the $\Delta\, AFC$

$FC = (a+b)\sin i$ and from $\Delta\, ACE$

$$CE = (a+b)\sin\theta$$

$$\therefore \quad FC + CE = (a+b)(\sin\theta + \sin i) \quad ...(i)$$



AB = a ; BC = b

Fig. 9.48

Equation (*i*) holds good if the beam is diffracted upwards. Fig. 9.49 illustrates the diffraction of the beam downwards. In this case the path difference

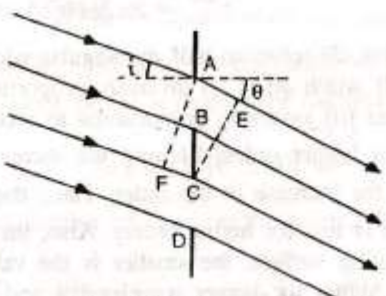
$$= (a+b)[\sin\theta - \sin i] \quad ...(ii)$$

For the *n* th primary maximum

$$(a+b)[\sin\theta_n + \sin i] = n\lambda \quad ...(iii)$$

or $$(a+b)\left[2\cdot\sin\frac{\theta_n+i}{2}\cdot\cos\frac{\theta_n-i}{2}\right] = n\lambda$$

or $$\sin\frac{\theta_n+i}{2} = \frac{n\lambda}{2(a+b)\cos\frac{\theta_n-i}{2}} \quad ...(iv)$$



AB = a ; BC = b

Fig. 9.49

The deviation of the diffraction beam $= \theta_n + i$

For the deviation $(\theta_n + i)$ to be minimum, $\sin\frac{\theta_n+i}{2}$ must be minimum. This is possible if the value of $\cos\frac{\theta_n-i}{2}$ is maximum *i.e.*,

$$\frac{\theta_n-i}{2} = 0 \quad \text{or} \quad \theta_n = i$$

Thus, the deviation produced in the diffracted beam is minimum when the angle of incidence is equal to the angle of diffraction. Let D_m be the angle of minimum deviation.

Then $$D_m = \theta_n + i$$

But $$\theta_n = i$$

$$\therefore \quad \theta_n = \frac{D_m}{2} \text{ and } i = \frac{D_m}{2}$$

$$\therefore \quad (a+b)\left(\sin\frac{D_m}{2} + \sin\frac{D_m}{2}\right) = n\lambda$$

or $$2(a+b)\sin\frac{D_m}{2} = n\lambda \quad ...(v)$$

Equation (*v*) refers to the principal maximum of the *n* th order for a wavelength λ.

Example 9.20. *A parallel beam of light of wavelength 5460 Å is incident at an angle of 30° on a plane transmission grating which has 6000 lines/cm. Find the highest order spectrum that can be observed.* [*Delhi (Hons.) 1984*]

$$(a+b)[\sin\theta_n + \sin i] = n\lambda$$

Here $$\theta_n = i$$

$$(a+b)(2\sin i) = n\lambda$$

Here $$(a+b) = \left(\frac{1}{6\times10^5}\right)\text{ m}$$

$$\lambda = 5460\text{ Å} = 5460\times10^{-10}\text{ m}$$

$$\sin 30 = 0.5$$

$$\therefore \quad n = \frac{(a+b)(2\sin i)}{\lambda} = \frac{1}{6\times10^5\times5460\times10^{-10}}$$

$$n = 3.05$$

or $$n = 3$$

9.37 ABSENT SPECTRA WITH A DIFFRACTION GRATING

In the equation $(a+b)\sin\theta = \lambda$, if $(a+b) < \lambda$, then $\sin\theta > 1$. But this is not possible. Hence the first order spectrum is absent. Similarly, the second, the third etc. order spectra will be absent if $(a+b) < 2\lambda$, $(a+b) < 3\lambda$ etc. In general, if $(a+b) < n\lambda$, then the *n* th order spectrum will be absent.

The condition for absent spectra can be obtained from the following consideration. For the *n* th order principal maximum

$$(a+b)\sin\theta_n = n\lambda \quad ...(i)$$

Further, if the value of a and θ_n are such that

$$a \sin \theta_n = \lambda \qquad ...(ii)$$

then, the effect of the wavefront from any particular slit will be zero. Considering each slit to be made up of two halves, the path difference between the secondary waves from the corresponding points will be $\frac{\lambda}{2}$ and they cancel one another's effect. If the two conditions given by equations (*i*) and (*ii*) are simultaneously satisfied, then dividing (*i*) by (*ii*)

$$\frac{(a+b)\sin\theta_n}{a\sin\theta_n} = \frac{n\lambda}{\lambda}$$

or

$$\frac{a+b}{a} = n \qquad ...(iii)$$

In equation (*iii*), the values of $n = 1, 2, 3$ etc. refer to the order of the principal maxima that are absent in the diffraction pattern.

(*i*) if $\frac{a+b}{a} = 1;\ b = 0$

In this case, the first order spectrum will be absent and the resultant diffraction pattern is similar to that due to single slit,

(*ii*) if $\frac{a+b}{a} = 2;\ a = b$

i.e., the width of the slit is equal to the width of the opaque spacing between any two consecutive slits. It this case, the second order spectrum will be absent.

9.38 OVERLAPPING OF SPECTRAL LINES

If the light incident on the grating surface consists of a large range of wavelengths, then the spectral lines of shorter wavelength and of higher order overlap on the spectral lines of longer wavelength and of lower order. Let the angle of diffraction θ be the same for (*i*) the spectral line of wavelength λ_1 in the first order, (*ii*) the spectral line of wavelength λ_2 in the second order and (*iii*) the spectral line of wavelength λ_3 in the third order. Then

$$(a+b)\sin\theta = 1\cdot\lambda_1 = 2\lambda_2 = 3\lambda_3 = \ldots$$

The red line of wavelength 7000 Å in the third order, the green line of wavelength 5250 Å in the fourth order and the violet line of wavelength 4200 Å in the fifth order are all formed at the same position of the screen because,

$$(a+b)\sin\theta = 3\times 7000\times 10^{-8}$$
$$= 4\times 5250\times 10^{-8}$$
$$= 5\times 4200\times 10^{-8}$$

Here $(a+b)$ is expressed in cm.

For the visible region of the spectrum, there is no overlapping of the spectral lines. The range of wavelengths for the visible part of the spectrum is 4000 Å to 7200 Å, Thus, the diffracting angle for the red end of the spectrum in the first order is less than the diffracting angle for the violet end of the spectrum in the second order. If, however, the observations are made with a photographic plate, the spectrum recorded may extend up to 2000 Å in the ultraviolet region. In this case, the spectral line corresponding to a wavelength of 4000 Å in the first order and a spectral line of wavelength 2000 Å in the second order overlap. Suitable fillers are used to absorb those wavelengths of the incident light which will overlap with the spectral lines in the region under investigation.

9.39 DETERMINATION OF WAVELENGTH OF A SPECTRAL LINE USING PLANE TRANSMISSION GRATING

In the laboratory, the grating spectrum of a given source of light is obtained by using a spectrometer. Initially all the adjustments of the spectrometer are made and it is adjusted for parallel rays by Schuster's method. The slit of the collimator is illuminated by monochromatic light (say light from a sodium lamp) and the position of the telescope is

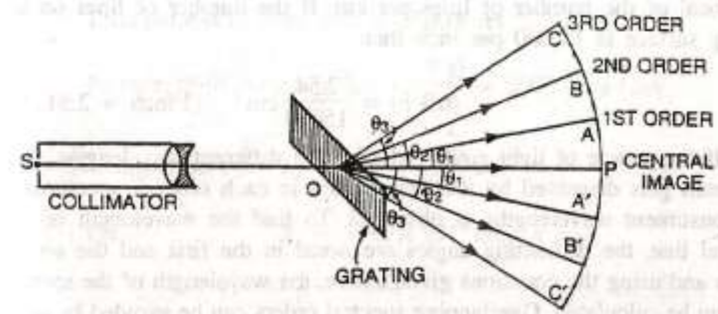


Fig. 9.50

adjusted such that the image of the slit is obtained at the position of the vertical cross-wire in the field of view of the telescope. Now the axes of the collimator and the telescope are in the same line. The position of the

$$(a+b)\sin\theta = n\lambda_1$$

$$(a+b)\times 0.5 = 3\times 5.4\times 10^{-7}$$

$$(a+b) = 3.24\times 10^{-6}\text{ m}$$

$$(a+b) = 3.24\times 10^{-4}\text{ cm}$$

Number of lines per cm,

$$N = \frac{1}{(a+b)}$$

$$N = \frac{1}{3.24\times 10^{-4}}$$

$$N = \mathbf{3086\ lines / cm}$$

9.40 DISPERSIVE POWER OF A GRATING

Dispersive power of a grating is defined as the ratio of the difference in the angle of diffraction of any two neighbouring spectral lines to the difference in wavelength between the two spectral lines. It can also be defined as the difference in the angle of diffraction per unit change in wavelength. The diffraction of the n th order principal maximum for a wavelength λ, is given by the equation,

$$(a+b)\sin\theta = n\lambda \qquad ...(i)$$

Differentiating this equation with respect to θ and λ [$(a+b)$ is constant and n is constant in a given order]

$$(a+b)\cos\theta\, d\theta = n\, d\lambda$$

or

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$$

or

$$\frac{d\theta}{d\lambda} = \frac{nN'}{\cos\theta} \qquad ...(ii)$$

In equation (*ii*) $\frac{d\theta}{d\lambda}$ is the dispersive power, n is the order of the spectrum, N' is the number of lines per cm of the grating surface and θ is the angle of diffraction for the n th order principal maximum of wavelength λ.

From equation (*ii*), it is clear, that the dispersive power of the grating is (1) directly proportional to the order of the spectrum, (2) directly proportional to the number of lines per cm and (3) inversely proportional to cos θ. Thus, the angular spacing of any two spectral lines is double in the second order spectrum in comparison to the first order.

Secondly, the angular dispersion of the lines is more with a grating having larger number of lines per cm. Thirdly, the angular dispersion is minimum when $\theta = 0$. If the value of θ is not large the value of cos θ can be taken as unity approximately and the influence of the factor cos θ in the equation (*ii*) can be neglected.

Neglecting the influence of cos θ, it is clear that the angular dispersion of any two spectral lines (in a particular order) is directly proportional to the difference in wavelength between the two spectral lines. A spectrum of this type is called a normal spectrum.

If the linear spacing of two spectral lines of wavelengths λ and $\lambda + d\lambda$ is dx in the focal plane of the telescope objective or the photographic plate, then

$$dx = f\, d\theta$$

where f is the focal length of the objective. The linear dispersion

$$\frac{dx}{d\lambda} = f\frac{d\theta}{d\lambda} = \frac{f\cdot nN'}{\cos\theta} \qquad ...(iii)$$

or

$$dx = \frac{fnN'}{\cos\theta}\cdot d\lambda$$

The linear dispersion is useful in studying the photographs of a spectrum.

9.41 PRISM AND GRATING SPECTRA

For dispersing a given beam of light and for studying the resultant spectrum, a diffraction grating is mostly used instead of a prism.

The following points give broadly the distinction between the spectra obtained with a grating and a prism.

(*i*) With a grating, a number of spectra of different orders can be obtained on the two sides of the central maximum whereas with a prism only one spectrum can be obtained.

(*ii*) The spectra obtained with a grating are comparatively pure than those with a prism.

(*iii*) Knowing the grating element $(a+b)$ and measuring the diffracting single, the wavelength of any spectral line can be measured accurately. But in the case of a prism the angles of deviation are not directly related to the wavelength of the spectral line. The angles of deviation are dependent on the refractive index of the material of the prism, which depends on the wavelength of light.

In equation (*iv*), if $t = 1$ cm, $\lambda = 5000$ Å and $\mu = 1.5$ then $n = 10{,}000$.

If the number of plates used is 40, the resolving power of the grating

$$= nN = 10{,}000 \times 40$$
$$= 4 \times 10^5$$

Thus, the resolving power of an echelon grating is very high and if the incident beam of light is not truly monochromatic, two nearby spectral lines will appear well resolved. The high resolving power of an echelon grating helps in the study of hyperfine structure *e.g.*, the splitting of spectral lines in Zeeman effect.

9.50 RESOLVING POWER OF OPTICAL INSTRUMENTS

The magnifying power of a telescope or a microscope depends on the focal length of the lenses used. By a proper choice of the lenses, it is possible to increase the size of the image, *i.e.*, the image subtends a large angle at the eye. But, it must be remembered that increase in the size of the image, beyond a certain limit does not necessarily mean gain in detail. This is the case even if the lenses are free from all aberrations, chromatic and monochromatic. There is always a limit to the useful magnification of an optical instrument. This is due to the fact that for a wave surface, the laws of geometrical optics do not hold good. In the preceding articles, concerning diffraction of light, it has been shown that the image of a point source is not a point but it is a diffraction pattern. With a circular aperture kept in the path of incident light, the diffraction pattern of a point source of light consists of a central bright disc surrounded by alternately dark and bright diffraction rings.

If the lens diameter or the size of the aperture is large, the diffraction pattern of a point source of light is small. If there are two nearby point sources, the diffraction discs of the two patterns may overlap and the two images may not be distinguished. An optical instrument like a telescope or a microscope is said to have resolved the two point sources when the two diffraction patterns are well separated from one another or when the diffraction patterns are small so that in both the cases, the two images are seen as separate ones. **The ability of an optical instrument, expressed in numerical measure, to resolve the images of two nearby points is termed as its resolving power.**

In the case of a prism or a grating spectrograph, the term resolving power is referred to the ability of the prism or grating to resolve two nearby spectral lines so that the two lines can be viewed or photographed as separate lines.

9.51 CRITERION FOR RESOLUTION ACCORDING TO LORD RAYLEIGH

To express the resolving power of an optical instrument as a numerical value, Lord Rayleigh proposed an arbitrary criterion. **According to him, two nearby images are said to be resolved if the position of the central maximum of one coincides with the first secondary minimum of the other and vice versa. The same criterion can be conveniently applied to calculate the resolving power of a telescope, microscope, grating, prism, etc.**

In Fig. 9.63, *A* and *B* are the central maxima of the diffraction patterns of two spectral lines of wavelengths λ_1 and λ_2. The difference in

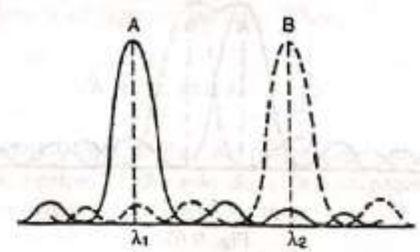


Fig. 9.63

the angle of diffraction is large and the two images can be seen as separate ones. The angle of diffraction corresponding to the central maximum of the image *B* is greater than the angle of diffraction corresponding to the first minimum at the right of *A*. Hence the two spectral lines will appear well resolved.

In Fig. 9.64 the central maxima corresponding to the wavelengths λ and $\lambda + d\lambda$ are very close. The angle of diffraction corresponding to

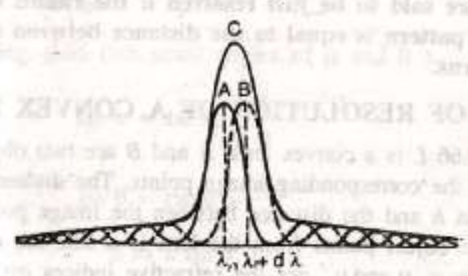


Fig. 9.64

the first minimum of A is greater than the angle of diffraction corresponding to the central maximum of B. Thus, The two images overlap and they cannot be distinguished as separate images. The resultant intensity curve gives a maximum as at C and the intensity of this maximum is higher than the individual intensities of A and B. Thus when the spectrograph is turned from A to B, the intensity increases, becomes maximum at C and then decreases. In this case, the two spectral lines are **not resolved.**

In Fig. 9.65, the position of the central maximum of A (wavelength λ) coincides with the position of the first minimum of B. (wavelength $\lambda + d\lambda$). Similarly, the position of the central maximum of B coincides with

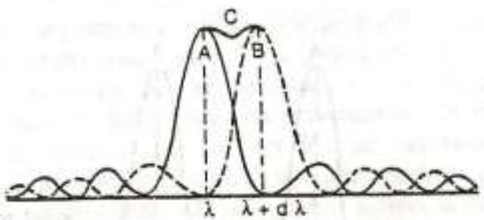


Fig. 9.65

the position of the first minimum of A. Further, the resultant intensity curve shows a **dip** at C *i.e.*, in the middle of the central maxima of A and B (Here, it is assumed that the two spectral lines are of the same intensity). The intensity at C is approximately 20% less than that at A or B. If a spectrograph is turned from the position corresponding to the central image of A to the one corresponding to the image of B, there is noticeable decrease in intensity between the two central maxima. The spectral lines can be distinguished from one another and according to Rayleigh they are said to be **just resolved. Rayleigh's condition can also be stated as follows. Two images are said to be just resolved if the radius of the central disc of either pattern is equal to the distance between the centers of the two patterns.**

9.52 LIMIT OF RESOLUTION OF A CONVEX LENS

In Fig. 9.66 L is a convex lens. A and B are two object points and A' and B' are the corresponding image points. The distance between the object points is h and the distance between the image points is h'. The distance of the object points from the lens is u and the distance of the image points is v. μ and μ' are the refractive indices of the object and image media. R is the radius of the aperture kept in front of the lens

(D is the diameter of the aperture). In the side figure, A' and B' are the centres of the central bright discs of the diffraction patterns of A and B.

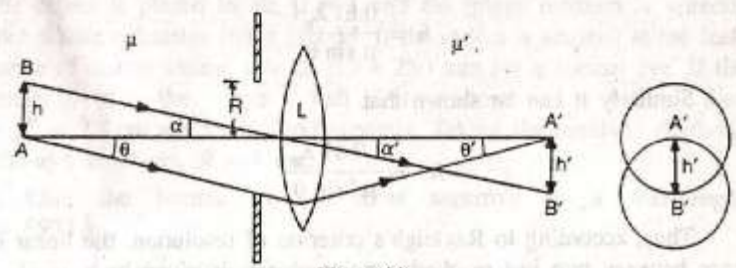


Fig. 9.66

Let λ and λ' be the wavelengths of light in the object and image media and λ the wavelength of light in vacuum. Then,

$$\lambda = \frac{\lambda_0}{\mu} \text{ and } \lambda' = \frac{\lambda_0}{\mu'}$$

According to Rayleigh, if the two images are just resolved, the distance between the centres of the two discs (h') is equal to the radius of either disc. If this condition is satisfied, then

$$\sin\alpha = \frac{1.22\lambda}{D} = \frac{1.22\lambda_0}{\mu D} = \frac{1.22\,\lambda_0}{2\mu R}$$

$$= \frac{0.61\,\lambda_0}{\mu R} \qquad ...(i)$$

Similarly, $$\sin\alpha' = \frac{0.61\lambda_0}{\mu' R} \qquad ...(ii)$$

From equation (*i*)

$$\mu R \sin\alpha = 0.61\lambda_0 \qquad ...(iii)$$

But, in Fig. 9.66 (for small angles of α and θ)

$$\sin\alpha = \tan\alpha = \frac{h}{u}$$

and $$\sin\theta = \tan\theta = \frac{R}{u}$$

or $$R = u\sin\theta$$

Substituting the values of $\sin\alpha$ and R in equation (*iii*)

$$h' = \frac{1}{1000}\text{ cm} = \frac{1}{100}\text{ mm approximately}$$

Also, $$\alpha = \sin\alpha = \frac{0.61\,\lambda_0}{\mu R}$$

$$= \frac{0.61 \times 5500 \times 10^{-8}}{1 \times 0.1}$$

$$= 0.00034 \text{ radian}$$

$$= 1 \text{ minute of an are (approximately)}$$

The value of h' ($= 10^{-3}$ cm) is approximately equal to the distance between the cones in the fovea and thus the retinal structure is strikingly in accordance with the limit of resolution of the eye. Further, two point objects appear just resolved if the angle subtended by them at the eye is 1 minute of an arc. If the diameter of the pupil of the eye is smaller than 2 mm the numerical aperture decreases and hence the value of h increases, i.e., two points will appear to be just resolved if the distance between the two is larger. Thus the resolving ability of the eye is decreased.

9.54 RESOLVING POWER OF A TELESCOPE

Let a be the diameter of the objective of the telescope (Fig. 9.68). Consider the incident ray of light from two neighbouring points of a distant object. The image of each point object is a Fraunhofer diffraction pattern.

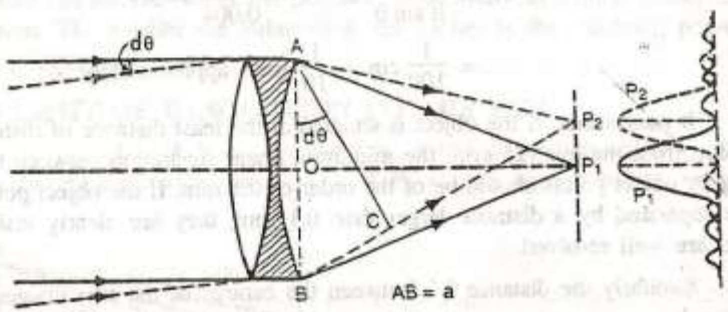


Fig. 9.68

Let P_1 and P_2 be the positions of the central maxima of the two images. According to Rayleigh, these two images are said to be resolved if the position of the central maximum of the second image coincides with the first minimum of the first image and *vice versa*. The path difference between the secondary waves travelling in the directions AP_1 and BP_1 is zero and hence they reinforce with one another at P_1. Similarly, all the secondary waves from the corresponding points between A and B will have zero path difference. Thus, P_1 corresponds to the position of the central maximum of the first image.

The secondary waves travelling in the directions AP_2 and BP_2 will meet at P_2 on the screen. Let the angle P_2AP_1 be $d\theta$. The path difference between the secondary waves travelling in the directions BP_2 and AP_2 is equal to BC (Fig. 9.68).

From the $\Delta\, ABC$,

$$BC = AB \sin d\theta = AB.d\theta = a\,.\,d\theta$$

(for small angles)

If this path difference $a\,.\,d\theta = \lambda$, the position of P_2 corresponds to the first minimum of the first image. But P_2 is also the position of the central maximum of the second image. Thus, Rayleigh's condition of resolution is satisfied if

$$a\,.\,d\theta = \lambda$$

or $$d\theta = \frac{\lambda}{a} \qquad ...(i)$$

The whole aperture AB can be considered to be made up of two halves AO and OB. The path difference between the secondary waves from the corresponding points in the two halves will be $\frac{\lambda}{2}$. All the secondary waves destructively interfere with one another and hence P_2 will be the first minimum of the first image. The equation $d\theta = \frac{\lambda}{a}$ holds good for rectangular apertures. For circular apertures, this equation, according to Airy, can be written as

$$d\theta = \frac{1.22\,\lambda}{a} \qquad ...(ii)$$

where λ is the wavelength of light and a is the aperture of the telescope objective. The aperture is equal to the diameter of the metal ring in which the objective lens is mounted. Here $d\theta$ refers to the **limit of resolution** of the telescope. **The reciprocal of $d\theta$ measures the resolving power of the telescope.**

$$\therefore \qquad \frac{1}{d\theta} = \frac{a}{1.22\,\lambda} \qquad ...(iii)$$

From equation (*iii*), it is clear that a telescope with large diameter of the objective has higher resolving power, $d\theta$ is equal to the angle subtended by the two distant object points at the objective.

Thus resolving power of a telescope can be defined as the reciprocal of the angular separation that two distant object points must have, so that their images will appear just resolved according to Rayleigh's criterion.

If f is the focal length of the telescope objective, then

$$d\theta = \frac{r}{f} = \frac{1.22\,\lambda}{a}$$

or $$r = \frac{1.22 f\lambda}{a} \qquad ...(iv)$$

where r is the radius of the central bright image. The diameter of the first dark ring is equal to the diameter of the central image. The central bright disc is called the Airy's disc.

From equation (*iv*), if the focal length of the objective is small, the wavelength is small and the aperture is large, then the radius of the central bright disc is small. The diffraction patterns will appear sharper and the angular separation between two just resolvable point objects will be smaller. Correspondingly, the resolving power of the telescope will be higher.

Let two distant stars subtend an angle of one second of an are at the objective of the telescope.

1 second of an are = 4.85×10^{-6} radian. Let the wavelength of light be 5500 Å. Then, the diameter of the objective required for just resolution can be calculated from the equation

$$d\theta = \frac{1.22\,\lambda}{a}$$

or $$a = \frac{1.22\lambda}{d\theta} = \frac{1.22 \times 5500 \times 10^{-8}}{4.85 \times 10^{-8}}$$

$$= 13.9 \text{ cm (approximately)}$$

The resolving power of a telescope increases with increase in the diameter of the objective. With the increase in the diameter of the objective, the effect of spherical aberration becomes appreciable. So, in the case of large telescope objectives, the central portion of the objective is covered with a stop so as to minimize the effect of spherical aberration. This, however, does not affect the resolving power of the telescope.

Example 9.40. *Find the separation of two points on the moon that can be resolved by a 500* cm *telescope. The distance of the moon is* 3.8×10^5 km. *The eye is most sensitive to light of wavelength 5500* Å. *(Nagpur 1974)*

The limit of resolution of a telescope is given by

$$d\theta = \frac{1.22\,\lambda}{a}$$

Here $\lambda = 5500 \times 10^{-8}$ cm, $a = 500$ cm

$$\therefore \quad d\theta = \frac{1.22 \times 5500 \times 10^{-8}}{500}$$

$$\therefore \quad d\theta = 13.42 \times 10^{-8} \text{ radian}$$

Let the distance between the two points be x

$$\therefore \quad d\theta = \frac{x}{R}$$

Here $R = 3.8 \times 10^{10}$ cm

$$x = R \cdot d\theta$$
$$= 3.8 \times 10^{10} \times 13.42 \times 10^{-8}$$
$$= 50.996 \times 10^2 \text{ cm}$$
$$= \mathbf{50.996 \text{ metres}}$$

Example 9.41. *Calculate the aperture of the objective of a telescope which may be used to resolve stars separated by* 4.88×10^{-6} *radian for light of wavelength 6000* Å

Here $\lambda = 6000$ Å $= 6 \times 10^{-5}$ cm, $\theta = 4.88 \times 10^{-6}$ radian

$D = ?$

$$\theta = \frac{1.22\,\lambda}{D}$$

or $$D = \frac{1.22\,\lambda}{\theta} = \frac{1.22 \times 6 \times 10^{-5}}{4.88 \times 10^{-8}} = \mathbf{15 \text{ cm}}$$

Example 9.42. *Two pin holes 1.5* mm *apart are placed in front of a source of light of wavelength* 5.5×10^{-5} cm *and seen through a telescope with its objective stopped down to a diameter of* 0.4 cm. *Find the maximum distance from the telescope at which the pin holes can be resolved.* *[Delhi, 1977]*

Here, $\lambda = 5.5 \times 10^{-5}$ cm

$a = 0.4$ cm

$$d\theta = \frac{1.22\,\lambda}{a}$$

Also $$d\theta = \frac{x}{d}$$

$\therefore$ Resolving power of a prism

$$= t\frac{d\mu}{d\lambda}$$

Thus, the resolving power of a prism is (*i*) directly proportional to the length of the base and (*ii*) rate of change of refractive index with respect to wavelength for that particular material. The expression for resolving power given above is applicable only to spectral lines of equal intensity. If two spectral lines are of different intensities, then the value of $d\lambda$ *i.e.*, the difference in wavelength between the two lines must be higher so that the two lines appear as separate ones.

Example 9.46. *The refractive indices of a glass prism for the C and F lines are 1.6545 and 1-6635 respectively. The wavelengths of these two lines in the solar spectrum are 6563 Å and 5270 Å respectively, Calculate the length of the base of a 60° prism which is capable of resolving sodium lines of wavelengths 5890 Å and 5896 Å.* (*Vikram University*)

$$\text{Resolving power} = \frac{\lambda}{d\lambda} = t\frac{d\mu}{d\lambda}$$

$$\text{Here} \quad \frac{d\mu}{d\lambda} = \frac{1.6635 - 1.6545}{(6563 - 5270)\ 10^{-8}}$$

$$\therefore \quad \frac{\lambda}{d\lambda} = t\left[\frac{1.6635 - 1.6545}{(6563 - 5270)\ 10^{-8}}\right]$$

$$\lambda = 5893 \times 10^{-8}\ \text{cm}, \quad d\lambda = (5896 - 5890)\ 10^{-8}$$

$$= 6 \times 10^{-8}\ \text{cm}$$

$$\therefore \quad t = \frac{5893 \times 10^{-8}}{6 \times 10^{-8}}\left[\frac{(6563 - 5270)\ 10^{-8}}{1.6635 - 1.6545}\right]$$

$$= \mathbf{1.41\ cm}$$

Example 9.47. *Calculate the minimum thickness of the base of a prism which will just resolve the D_1 and D_2 lines of sodium. Given μ for wavelength 6563 Å = 1.6545 and for wavelength 5270 Å = 1.6635.* [*Bombay*]

In a prism,

$$\text{Resolving power,}\ \frac{\lambda}{d\lambda} = t\frac{d\mu}{d\lambda}$$

$$\text{Here} \quad \frac{d\mu}{d\lambda} = \left[\frac{1.6635 - 1.6545}{(6563 - 5270) \times 10^{-8}}\right] = \left(\frac{0.0090}{1293 \times 10^{-8}}\right)$$

$$\text{and} \quad \frac{\lambda}{d\lambda} = \frac{5893 \times 10^{-8}}{6 \times 10^{-8}} = \frac{5893}{6}$$

$$\therefore \quad t = \frac{\left(\dfrac{\lambda}{d\lambda}\right)}{\left(\dfrac{d\mu}{d\lambda}\right)}$$

$$\therefore \quad t = \left(\frac{5893 \times 1293 \times 10^{-8}}{6 \times 0.0090}\right)\ \text{cm}$$

$$t = \mathbf{1.41\ cm}$$

9.59 RESOLVING POWER OF A PLANE DIFFRACTION GRATING

The resolving power of a grating is defined as the ratio of the wavelength of any spectral line to the difference in wavelength between this line and a neighbouring line such that the two lines appear to be just resolved. Thus, the resolving power of a grating appear to be just resolved. Thus, the resolving power of a grating

$$= \frac{\lambda}{d\lambda}$$

In Fig. 9.74, *XY* is the grating surface and *MN* is the field of view of the telescope, P_1 is the *n* th primary maximum of a spectral line of wavelength λ at an angle of diffraction θ_n. P_2 is the *n* th primary maximum

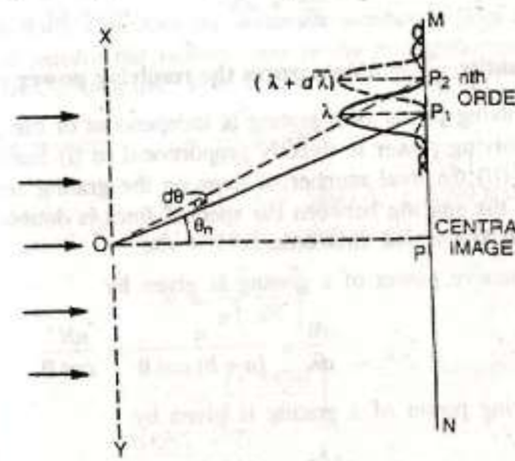


Fig. 9.74

of a second spectral line of wavelength $\lambda + d\lambda$ at a diffracting angle $\theta_n + d\theta$. P_1 and P_2 are the spectral lines in the n th order. These two spectral lines according to Rayleigh, will appear resolved if the position of P_2 also corresponds to the first minimum of P_1.

The direction of the n th primary maximum for a wavelength λ is given by

$$(a+b)\sin\theta_n = n\lambda \qquad ...(i)$$

The direction of the n th primary maximum for a wavelength $(\lambda + d\lambda)$ is given by

$$(a+b)\sin(\theta_n + d\theta) = n(\lambda + d\lambda) \qquad ...(ii)$$

The two lines will appear just resolved if the angle of diffraction $(\theta_n + d\theta)$ also corresponds to the direction of the first secondary minimum after the n th primary maximum at P_1 (corresponding to wavelength λ). This is possible if the extra path difference introduced is $\frac{\lambda}{N}$ Where N is the total number of lines on the grating surface.

$$\therefore \quad (a+b)\sin(\theta_n + d\theta) = n\lambda + \frac{\lambda}{N} \qquad ...(iii)$$

Equating the right hand sides of equations (*ii*) and (*iii*)

$$n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N}; \quad n, d\lambda = \frac{\lambda}{N}$$

$$\frac{\lambda}{d\lambda} = nN \qquad ...(iv)$$

The quantity $\frac{\lambda}{d\lambda} = nN$ measures the resolving power of a grating. Thus, the resolving power of a grating is independent of the grating constant. The resolving power is directly proportional to (*i*) the order of the spectrum and (*ii*) the total number of lines on the grating surface. For a given grating, the spacing between the spectral lines is double in the second order than that in the first order.

The dispersive power of a grating is given by

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta} = \frac{nN'}{\cos\theta}$$

and the resolving power of a grating is given by

$$\frac{\lambda}{d\lambda} = nN$$

where n is the order of the spectrum, N is the total number of lines of the grating. N' is the number of lines per cm on the grating surface. Here, θ gives the direction of the n th principal maximum corresponding to a wavelength λ. From the above equation, it is clear, that the dispersive power increases with increase in the number of lines per cm and the resolving power increases, with increase in the total number of lines on the grating surface (*i.e.*, the width of the grating surface). If N' is the same for two gratings, the dispersive power will be the same in the two cases but the one with larger width of the grating surface produces higher resolution of the spectral lines. With a grating having large width of the grating surface, the spectral lines are sharp and narrow.

High dispersive power refers to wide separation of the spectral lines whereas high resolving power refers to the ability of the instrument to show nearby spectral lines as separate ones.

Example 9.48. *What should be the minimum number of lines in a grating which will just resolve in the second order the lines whose wavelengths are 5890 Å and 5896 Å ?* (*Agra*)

Resolving power $= \frac{\lambda}{d\lambda} = nN$

Here, $n = 2$, $\lambda = 5890$ Å, $d\lambda = 5896 - 5890 = 6$ Å

$$\therefore \quad \frac{5890}{6} = 2N$$

or $$N = \frac{5890}{6 \times 2} = \textbf{491 approximately.}$$

Example 9.49. *Calculate the minimum number of lines in a grating which will just resolve the sodium lines in the first order spectrum. The wavelengths are 5890 Å and 5896 Å.* (*Delhi*)

Resolving power $= \frac{\lambda}{d\lambda} = nN$

Here, $n = 1$, $\lambda = 5890$ Å, $= 5890 \times 10^{-8}$ cm

$$d\lambda = 5896 - 5890 = 6 \text{ Å} = 6 \times 10^{-8} \text{ cm}$$

$$N = \frac{1}{n}\left[\frac{\lambda}{d\lambda}\right]$$

$$= \frac{1}{1}\left[\frac{5890}{6}\right]$$

or $$n = \textbf{982 approximately.}$$